

2006

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**A general queuing network model to optimize refurbishing policies  
for returned products**

by

**Jumpol Vorasayan**

A dissertation submitted to the graduate faculty  
in partial fulfillment of the requirements for the degree of  
DOCTOR OF PHILOSOPHY

Major: Industrial Engineering

Program of Study Committee:  
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Ames, Iowa

2006

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## TABLE OF CONTENTS

<b>LIST OF FIGURES.....</b>	<b>v</b>
<b>LIST OF TABLES.....</b>	<b>vii</b>
<b>ACKNOWLEDGEMENTS.....</b>	<b>viii</b>
<b>CHAPTER 1. INTRODUCTION.....</b>	<b>1</b>
1.1 Statement of Problem.....	1
1.2 Research Objective.....	5
1.3 Organization of Research.....	6
<b>CHAPTER 2. LITERATURE REVIEW .....</b>	<b>8</b>
<b>CHAPTER 3. MODEL FORMULATION AND OPTIMALITY</b>	
<b>CONDITIONS .....</b>	<b>13</b>
3.1 The Demand Function.....	16
3.2 The Queueing Network Model.....	20
3.2.1 General Queueing Network Model .....	23
3.2.2 Jackson Open Queueing Network Model .....	32
3.3 The Model Formulation and its Optimality Conditions.....	33
3.3.1 General Queueing Network Model .....	34
3.3.2 Jackson Open Queueing Network Model .....	41
<b>CHAPTER 4. NUMERICAL EXAMPLES .....</b>	<b>43</b>
4.1 Jackson Open Queueing Network Model .....	43

4.1.1	Cost and Quality of Refurbished Products .....	47
4.1.2	Price and Backorder Cost for New Products .....	47
4.1.3	Sensitivity of the Optimal Quantity of Refurbished Products.....	53
4.2	General Queueing Network Model .....	55
4.2.1	Effect of Variability.....	55
4.2.2	Evaluating and Sorting Returned Products.....	64
4.2.3	SEPT Policy.....	69
4.2.4	The Number of Times Products Can Be Refurbished .....	70
<b>CHAPTER 5. CONCLUSIONS AND FUTURE RESEARCH.....</b>		<b>76</b>
<b>APPENDIX.....</b>		<b>79</b>
<b>REFERENCES.....</b>		<b>91</b>

## LIST OF FIGURES

FIGURE 1. Dell online store for new and refurbished Dell notebooks.....	15
FIGURE 2. Ranger of prices with respect to refurbished product quality that govern demand for new and refurbished products.....	19
FIGURE 3. The open queueing network .....	20
FIGURE 4. The open queueing network for $\eta = 1$ .....	23
FIGURE 5. Two feasible regions depending on the relationship between demand and manufacturing capacity for new products. ....	40
FIGURE 6 Total profit for different values of $p_{mr}$ for $P_{new} = 0.45$ , $c_{45} = 0.06$ , $h_1 = 0.0001$ and $\delta = 0.82, 0.86$ and $0.90$ .....	46
FIGURE 7. Total profit, $P_{ref}^*$ and $p_{mr}^*$ for different values of $\delta$ and $c_{45}$ at $P_{new} = 0.45$ and $h_1 =$ $0.0001$ .....	50
FIGURE 8. Feasible regions with two different sets of constraints depending on $P_{new}$ .....	50
FIGURE 9. The optimal total profit for different $P_{new}$ , $h_1$ , $c_{45}$ and $\delta$ . ....	51
FIGURE 10. The optimal price for refurbished products for different $P_{new}$ , $h_1$ , $c_{45}$ and $\delta$ ....	51
FIGURE 11. The optimal proportion of returns to refurbish for different $P_{new}$ , $h_1$ , $c_{45}$ and $\delta$ . .....	51
FIGURE 12. The response of $p_{mr}^*$ to different $P_{new}$ , $h_1$ , $\delta$ and $c_{45}$ .....	55
FIGURE 13. Total profit and price of refurbished products from Jackson open queueing and general open queueing models at $\delta = 0.9$ . ....	58

Figure 14. Total profit, $P_{ref}^*$ and $p_{mr}^*$ from the Jackson open queueing and the general open queueing networks for different quality levels.....	60
Figure 15. Effect of variability of station 1 service times on $\hat{p}_{mr}^*$ , $P_{ref}^*$ and total profit for different levels of perceived refurbished item quality .....	62
Figure 16. Effect of variability of station 4 service times on $p_{mr}^*$ , $P_{ref}^*$ and total profit for different levels of perceived refurbished item quality. ....	63
Figure 17. $p_{mr}^*$ , $P_{ref}^*$ and total profit from the general open queueing model with sorted and unsorted grades of returned products for different quality levels of refurbished products. ....	68
Figure 18. Total profit at $P_{ref}^*$ with different quantities of refurbished products from Unsorted and Sorted cases at $\delta = 0.9$ .....	69
Figure 19. Percent difference in inventory cost at station 4 achieved by SEPT rather than FCFS policy .....	70
Figure 20. Total profit, $P_{ref}^*$ , $p_{34}^*$ and $\lambda_4$ from the general open queueing network with different numbers of times that products can be refurbished.....	75



## LIST OF TABLES

TABLE 1. Reasons for product returns.....	2
TABLE 2. Product types and routings. ....	22
TABLE 3. Breakdown of costs in $c_{ij}$ .....	34
TABLE 4. Comparing local optima to find the global optimum. ....	46
TABLE 5. The effect of parameters $P_{new}$ , $h_1$ , $\delta$ and $c_{45}$ on total profit, $p_{mr}^*$ , and $P_{ref}^*$ .....	53
TABLE 6. The effect of variability at station 1 to 4 on total profit, $p_{mr}^*$ , and $P_{ref}^*$ .....	61

## **ACKNOWLEDGEMENTS**

I would like to express my sincerest appreciation to Dr. Sarah M. Ryan, my major professor, for the greatest knowledge, tremendous effort and unforgettable kindness given to her advisee. My gratitude also goes to Dr. Doug Gemmill, Dr. Jo Min, Dr. Sigurdur Olafsson and Dr. Soumenda Lahiri whose guidance and inspiration greatly influenced this work. I would like to thank all my friends and colleagues in IMSE department, Thai Student Association, and Iowa State University for their friendship and all memorable experiences. Many thanks also go to Jeffery Doran for being a friend, English and math tutor at the same time. Most of all, my deepest gratitude goes to Pornchai and Jintana Vorasayan, my parents, Jiraporn, my sister, and Jittapol, my brother, who always love and support me unconditionally.

## CHAPTER 1. INTRODUCTION

### 1.1 Statement of Problem

Over \$100 billion worth of products are returned from customers to retailers annually (Stock, Speh and Shear, 2002). The reasons for and time scales of these enormous returns are summarized in Table 1 (Silva, 2004 and Souza et al., 2005). Other than at the end of life, products are returned relatively soon after distribution. Dowling (1999) shows that up to 35 percent of new products are returned before the end of their life cycle. The value of these returns is considerable since they still preserve features and technologies of new products that are currently for sale. When returns come to manufacturers, the right decision has to be made to manage these returns profitably. Depending on their quality and the manufacturer's policy, some returns even qualify to be sold again as new products to regain the total margin. Products that have been used or have some defects will be either refurbished then resold whole or dismantled into parts that are either kept for service or sold.

**TABLE 1. Reasons for product returns**

<b>Reasons of product returns</b>	<b>Description</b>	<b>Length of time (approximately)</b>
Customer satisfaction	The quality of product does not meet the customer's expectation. This category also includes miscellaneous reasons such as customers cannot use products, find the better price, over ordered or feel remorse.	Return period (30 days)
Evaluation product	Products which were reviewed and tested by editors or vendors.	Evaluation period (30 days)
Shipping damage	Products cannot be sold as new when their containers are damaged.	Shipping period (< 7 days)
Defective	The product cannot perform functions as described.	Warranty period (1 year)
End of lease	The product is returned after the end of the lease.	Lease period (varied)
End of life	The product is collected after it has been disposed.	Life cycle of product (varied)

Refurbished products are those that have been verified by the manufacturer to be as functional as new products. White and Naghibi (1998) described the refurbishment process as complying with the highest standards and giving careful attention to both the interior and the exterior of the product. Electronic products are subjected to rigorous electrical testing to ensure they meet all original manufacturing specifications. Examples of products that qualify for refurbishment are consumer-returned products, off-lease products, products with shipping damage, and over stocks (Silva, 2004 and Souza et al., 2005). Nevertheless, different companies may have different policies for their refurbishment process. We provide definitions of refurbished products from Dell, Apple and Olympus as follows;

Dell<sup>1</sup>: *“All of Dell's new computers come with a 21 day total satisfaction policy. As a result of this policy, some computers are returned to Dell. We disassemble and rebuild returned systems to original factory specifications, then put the systems through a rigorous testing program to ensure Dell quality. We even use brand new boxes for packaging! As a result of our fantastic refurbishment process, Dell refurbished systems include the same limited warranty as new systems. In addition to our refurbished systems, we offer a limited number of factory sealed storage products that have been returned to Dell unopened. These unopened (factory-sealed) units do not go through the testing or rebuilding process that refurbished systems are subject to. They are sold and shipped in their original factory tested condition. We provide the same limited warranty on these factory sealed storage systems as we provide on Dell refurbished systems.”*

Apple<sup>2</sup>: *“Apple Certified Refurbished products are pre-owned Apple products that undergo Apple's stringent refurbishment process prior to being offered for sale. All Apple Certified Refurbished products are covered by Apple's One-Year Limited Warranty.”*

Olympus<sup>3</sup>: *“Olympus defines reconditioned products as returned product that has been tested, inspected, and certified to be functional. Though some of our reconditioned products may have some minor cosmetic imperfections, they are operationally perfect and include the same box components that a new product would have. Reconditioned products are shipped with a 90 day warranty covering all defects, and Emporium purchases can be returned within 30 days for a full refund.”*

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<sup>1</sup> [www.dell.com](http://www.dell.com) (viewed 11/01/05)

<sup>2</sup> [www.apple.com](http://www.apple.com) (viewed 11/01/05)

<sup>3</sup> [www.olympusamerica.com](http://www.olympusamerica.com) (viewed 11/01/05)

From the consumer perspective, buying refurbished products is an economical way to obtain goods that perform as well as new products. For the manufacturer, refurbished products broaden the market by increasing product differentiation to induce consumers in the segment that is not willing to pay full price to purchase refurbished products for less. However, there may be an overlap between the markets for new and refurbished products. Consumers in this overlap market will choose between new or the refurbished product based on price and perceived quality.

In this research, we study an original equipment manufacturer who assembles and sells both new and refurbished products directly to consumers, such as a computer maker who sells products online. New products are produced to order, but for a variety of reasons, some of them are returned soon after sale for a refund. One choice is whether to refurbish returns and offer them for sale, and if so, how many. Unlike the market for new products, where the manufacturer is a price-taker, we assume that because the manufacturer would be the only source for certified refurbished products, it is able also to choose the price at which to offer its inventory of refurbished items. This price must be chosen with care because it will play an important role in determining the demand for both new and refurbished products. In addition, there may be substantial costs associated with refurbishing products and holding them in inventory. Therefore, both the price and the quantity of refurbished products may have significant impacts on the manufacturer's total profit.

## 1.2 Research Objective

When products have been returned before their end of life, they still preserve most of their qualities and can be resold again with the appropriate refurbishment. However, introducing refurbished products to the market may cannibalize the demand of new products since consumers may choose to buy either one. This research tries to find the optimal combination of two variables: the proportion of product returns to be refurbished and its corresponding price that maximize the total profit.

In order to find the optimal price and quantity of refurbished products, we use an open queueing network model to simulate the reverse logistics scheme. The closed loop supply chain model incorporates the assembly of new products, return, evaluation, refurbishment and resale processes. The costs incurred in this supply chain are inventory, transportation, production, and handling costs. The objective function is the total profit which can be calculated from the difference between revenues from selling new, refurbished and dismantled products, and all costs. We model the total profit as a function of the price of refurbished products and the proportion of returned products that will be refurbished. In the queueing network model, this proportion is modeled as a probability that a returned product is sent to the refurbishment station. If this probability equals zero, none of returned products will be refurbished. When the probability equals one, all returned products will be refurbished.

A general open queueing network with general service time distributions under a two-moment approximation is used to find the optimal solution. With this queueing network, we can limit the number of times an item can be refurbished and set a priority rule at the

refurbishment station. We also develop a special case of the general queueing network where all service times are exponential, products can be refurbished for an unlimited number of times and no priority scheme is used. This special model is a Jackson open queueing network which can be solved exactly in much less computation time. Because it requires less time to solve, a large number of numerical experiments are done with the Jackson network to give an insight into the model (Vorasayan and Ryan, 2006). The general service time studies focus on parameters involved in the refurbishing process, e.g., the number of times products can be refurbished, and the priorities assigned to different grades of returned products.

The optimal price and quantity of refurbished products vary depending on factors such as the cost and quality of refurbished products, the price and backorder cost of new products, and the number of times products can be refurbished. In this research, we vary these parameters to find their effect on the optimal price and quantity of refurbished products. Moreover, we also study the sensitivity of the optimal value of decision variables, the effect of variability and the advantage of sorting and prioritizing returned products according to their grades and return histories.

### **1.3 Organization of Research**

The remainder of this research is organized as follows: Chapter 2 contains a literature review of previous approaches to apply queueing networks in closed-loop supply chains. Chapter 3 provides the mathematical model and Chapter 4 shows numerical examples that illustrate the implementation of the model and its results. Chapter 5 concludes with a description of future work. An Appendix contains proofs of the feasible region's convexity,



details of the Karush-Kuhn-Tucker optimality conditions and a proof of the concavity of total profit with respect to the price of new products.

## CHAPTER 2. LITERATURE REVIEW

The management of item returns has been studied in a variety of models. Fleischmann et al. (1997) reviews the quantitative models for reverse logistics in three main areas: distribution planning, inventory control, and production planning. For inventory control in remanufacturing, Van der Laan and Salomon (1997) develop a model to control inventory under push and pull policies. They also study how much the system cost will be affected when a disposal operation is implemented along with remanufacturing. By a numerical example, they show that disposal operations result in lower costs by reducing variability in the system inventories. A different approach to finding the optimal  $(s, Q)$  inventory control policy is developed by Van der Laan et al. (1996) and Fleischmann et al. (2002) with the assumption that the item can be reused with and without prior treatment, respectively. The item returns and demand rates are assumed to follow independent Poisson processes.

In areas of distribution and production, Fasano et al. (2002) use an optimization tool to determine if end-of-life IBM equipment should be sold as whole or dismantled for service parts. They assume that demand for refurbished products in a particular time period is limited, and that a machine will be refurbished only when there is demand and potential positive net revenue that result in a specified profit margin. Savaskan et al. (2004) address the situation where the retailer is a more effective channel for collecting used products than either the manufacturer herself or a third party. Ferguson et al. (2006) show that coordination between the manufacturer and retailer to reduce fault failure returns (non-defective-returned products) significantly improves profit to both parties.

The study on market segmentation where different qualities of products are offered to heterogeneous consumers can be found in Mussa and Rosen (1978) and Moorthy (1984). Levinthal and Purohit (1989) develop a two-period model to examine the optimal sales strategy for a monopolist manufacturer who introduces new products to the market where old products are still for sale. Purohit (1992) shows that price of cars in the secondary market responds to changes in the primary market for new cars. The result also suggests that depreciation of used products is strongly affected by the model improvements from new products. Villas-Boas (1998) shows that increasing the quality differences in the product lines helps coordination between manufacturer and retailer to sell products to the right consumer segments. Arunkundram and Sundararajan (1998) consider the competition of used and new products in the electronic secondary markets. The paper shows the situation where the secondary market benefits the profitability of new products sales. We construct our demand function similarly to Arunkundram and Sundararajan (1998) by considering competition between refurbished and new instead of used and new products.

Competition arises when remanufactured products enter the market. Majumder and Groenevelt (2001) study the competition in remanufacturing between an original equipment manufacturer and a local remanufacturer under a two-period model. In the first period, only new products are made by the manufacturer. In the second period additional to new products, remanufactured products are made by both parties from returned products from the first period. The game theoretic model finds the Nash equilibrium of the price and quantity for both competitors in four scenarios. Ferrer and Swaminathan (2005) extend the previous work by developing a multi-period model to find the Nash equilibrium for the duopoly case. Ferguson and Toktay (2006) examine competition for remanufactured products in more

detail, exploring strategies by which the manufacturer can exploit its access to used products to ward off third party remanufacturers. In more closely related work, Debo et al. (2005a) study a monopolist's decision of whether to produce a remanufacturable product, where competition with third party remanufacturers may exist. The additional cost to make a product remanufacturable may be worthwhile if enough customers value the remanufactured product highly, but competition reduces the optimal level of remanufacturability. They also expose the role of new products as a source for products to be remanufactured.

Categorizing the quality of returns is another important aspect of a remanufacturing environment. Klausner et al. (1998) show the potential cost savings from attaching a small device to the motor for recording the history of power tool usage. The studies of remanufacturing used cell phones in Souza et al. (2001), and Guide et al. (2003) also consider multiple grade levels. Each grade level requires a different processing time and cost for remanufacturing. Aras et al. (2004) shows the situation where prioritizing according to the grade of returned products is the most cost effective in the hybrid manufacturing and remanufacturing systems.

Other recent related works in closed-loop supply chains are as follows: Guide et al. (2003) propose an economic analysis for finding the optimal acquisition and selling prices, along with quantity of used product acquisitions, for a remanufacturer in the cellular telephone industry. Debo et al. (2005b) study the joint diffusion of new and remanufactured products in the market by using the Bass diffusion model. The results show several phenomena of sales paths that differ according to factors such as the distribution of resident time of new and remanufactured products, manufacturing capacity, remanufacturability level and market penetration rate.

Several types of queueing models have been applied in the remanufacturing environment. Guide and Gupta (1999) develop queueing models to estimate the flow time of remanufacturing products flowing through three different segments: disassembly, remanufacturing and reassembly. The model yields results fairly similar to those from simulation. Toktay et al. (2000) construct a queueing network to simulate the entire supply chain of a single-use camera. The optimization model minimizes the costs of procurement, inventory, and lost sales for different policies. Bayindir et al. (2003) find the optimal probability of return for items sold. They assume that the returns are controllable and the manufacturer has infinite capacity. Unlike in our model, the consumers are indifferent between the new and remanufactured products. Souza et al. (2002) simulate the remanufacturing facility as a  $GI/G/1$  queueing network. The product returns have different grades that require different processing times. The model finds the optimal product mix for remanufacturing to maximize profit. Ketzenberg and Souza (2003) compare two configurations of a remanufacturing process, mixed and parallel lines, in several scenarios by using a  $GI/G/c$  queueing network. Souza et al. (2006) analyze and suggest the appropriate supply chain for commercial product returns of products with different rates of decay in price.

The performance approximation of the general open queueing network model by the parametric-decomposition method was first proposed by Reiser and Kobayashi (1974). It has been used and continuously developed by Servick et al. (1977), Chandy and Sauer (1978), Kuehn (1979), Shanthikumar and Buzacott (1981), and Whitt (1983). Bitran and Tirupati (1988) observe that the decomposition methods previously studied were not accurate in multiple-class, low variability, deterministic routing situations and they proposed an

improved equation to approximate the variability of the flow from one node to another. Segal and Whitt (1989) suggest the convex combination of approximate equations in Whitt (1983) and Bitran and Tirupati (1988) when the network has both probabilistic and deterministic routings. The approximation of the departure process model for deterministic routing in the Bitran and Tirupati model was extended by Whitt (1994) to obtain more effective results.

Although many queueing networks have been used in the reverse logistics literature, none of them are used in the market segmentation area. In this research, the queueing model allows us to include the supply chain of products together with their market segmentation. It permits the accounting of costs associated with the refurbishing policy throughout the system including transportation, handling and holding inventories of rapidly depreciable products. This model well simulates the supply chain of manufacturers who manufacture new products to order and refurbish products to stock. This supply chain configuration has been extensively applied in the personal computer and automotive industries. Unlike the previous market segmentation literature, our model includes variability in the times for which products are manufactured, refurbished, stored in inventory and used by consumers. Moreover, the model can specify the number of times products can be refurbished to be any number rather than limited to once (Debo et. al 2005a) or unlimited (Toktay et al. 2000, Souza et al. 2006). The optimal quantity of refurbished products can be any proportion of returned products, not limited to none or all of the items. We show how the optimal price and quantity of refurbished products are affected by several factors in the system.

## CHAPTER 3. MODEL FORMULATION AND OPTIMALITY CONDITIONS

This research is primarily motivated by manufacturers who produce and sell products via an online store, e.g., Dell, Apple, or Gateway Computers. New computers are assembled according to specifications customized from consumer orders. Consumers can customize CPU speed, hard drive and memory size along with other hardware and software to satisfy their needs. This policy is called Make-to-Order. Once these consumers receive products, they can use and either keep or return them within an evaluation period. These returned products are evaluated and scheduled to be dismantled or refurbished. Products that have been refurbished are kept to be resold at a reduced price in the online store for refurbished products. The policy for storing items to satisfy future demand called a Make-to-Stock policy. Because the amount of time during which these electronic products have been sold, returned, and refurbished is very short compared to their life cycle, we can often see that refurbished products are essentially the same model and specification as new products. Figure 1 shows the new and refurbished product online store of Dell for (a) new and (b) refurbished notebooks<sup>4</sup>.

In this chapter, we first describe assumptions and the demand functions for new and refurbished products. Next, the supply chain is simulated as an open queueing network. Finally, we present a nonlinear program for maximizing total profit and outline its optimality conditions.

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<sup>4</sup> Screenshot from Dell store for Inspiron notebook on 11/01/05; this information is constantly updated and subjected to change. For more information visit [www.dell.com](http://www.dell.com).

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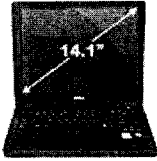
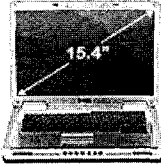

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<b>Processor</b> Mobile Intel® Celeron® M Processor 350 at 1.30 GHz (1MB Cache, 400MHz FSB)	<b>Processor</b> Up to Intel® Pentium® M 755 Processor (1.70GHz, 2MB Cache, 400 MHz)	<b>Processor</b> Up to Intel® Pentium® M Processor 730 (1.60 GHz, 2MB Cache, 533 MHz FSB) with Intel Centrino™ Mobile Technology	<b>Processor</b> Up to Intel® Pentium® M Processor 755 (2 GHz, 2MB Cache, 400 MHz FSB) with Intel Centrino™ Mobile Technology
<b>Operating Systems</b> Genuine Windows® XP Home	<b>Operating Systems</b> Genuine Windows® XP Professional Edition Genuine Windows® XP Home Edition	<b>Operating Systems</b> Genuine Windows® XP Professional Edition Genuine Windows® XP Home Edition	<b>Operating Systems</b> Genuine Windows® XP Professional Edition Genuine Windows® XP Home Edition
<b>Screen</b> Up to 15" XGA TFT display	<b>Screen</b> Up to 15" XGA TFT display	<b>Screen</b> Up to 15.4" Wide Screen UltraSharp™ WUXGA display	<b>Screen</b> Up to 14.1" TFT SXGA+ display

(b) Dell online store for refurbished notebooks.

FIGURE 1. Dell online store for new and refurbished Dell notebooks.

### 3.1 The Demand Function

Demands for new and refurbished products are interdependent and can be described as functions of their prices and the perceived quality of refurbished products. We assume the price of new products is an exogenous constant, as in a market where different brands of products have similar performance, so that a small change in price may cause a significant change in market share. On the other hand, as the sole source of manufacturer-certified refurbished products, the producer can control both their price and their supply. We focus on the market where consumers have declared interest in a specific brand and model of products but are still deciding whether to buy either new or refurbished or none at all. That is, we explicitly model only the internal competition between the new and the refurbished products. This competition can be viewed as imperfect or monopolistic since the products are similar but one is still not a perfect substitute for the other (Nichols and Reynolds, 1971). The demand model is similar to those of Arunkundram and Sundararajan (1998) and Debo et al. (2005a). Although the price of new products is not a decision variable, it is varied in the Jackson network case to study its effect on the objective function and decision variables.

The market size of these consumers in a given study period is normalized to one. In this market, the valuations of consumers are uniformly distributed from 0 to 1. The consumer's willingness to pay or valuation of a product is directly proportional to its perceived quality. If a consumer values the new product at  $v$ , then that consumer values the refurbished product at  $\delta v$ . The parameter  $\delta$  is the perceived quality factor of refurbished products compared to new products,  $0 < \delta < 1$ . This implies that refurbished products will not have the same perceived quality as new products although their performances are

indistinguishable. The perceived quality includes many attributes such as technical specification, warranty period and physical appearance that vary as described in section 1.1 depending on the refurbishment policy. We are particularly interested in this parameter since it depends not only on the consumer but also on company policy. We assume  $\delta$  is a constant parameter and is independent of the grades of returned products. That is, although returned products may have different grades, e.g., unopened, defective, used etc., we assume they can be refurbished into the same quality.

The price of new products ( $P_{new}$ ) is scaled down to the same scale as the consumer's valuation ranging from 0 to 1. The value  $P_{new} = 0$  corresponds to the minimum value of  $v$  and  $P_{new} = 1$  is the maximum possible value of  $v$ . When we consider only new products in the market, if  $P_{new} = 1$ , no consumers in the market will buy the new product since no one has valuation higher than the price of new products. In contrast, when  $P_{new} = 0$ , all consumers in the market are willing to buy new products. The scaled price of refurbished products ( $P_{ref}$ ) ranges from 0 to  $P_{new}$ . A consumer will buy the product that gives him the higher surplus, which is the difference between his valuation and the product price. A consumer with valuation  $v$  will choose to buy the new product when  $v - P_{new} \geq 0$  and  $v - P_{new} \geq \delta v - P_{ref}$ , i.e.,  $v \geq \frac{P_{new} - P_{ref}}{1 - \delta}$ . On the other hand, he will buy the refurbished

product when  $\delta v - P_{ref} \geq 0$  and  $v - P_{new} < \delta v - P_{ref}$ , i.e.,  $\frac{P_{ref}}{\delta} \leq v < \frac{P_{new} - P_{ref}}{1 - \delta}$ . If

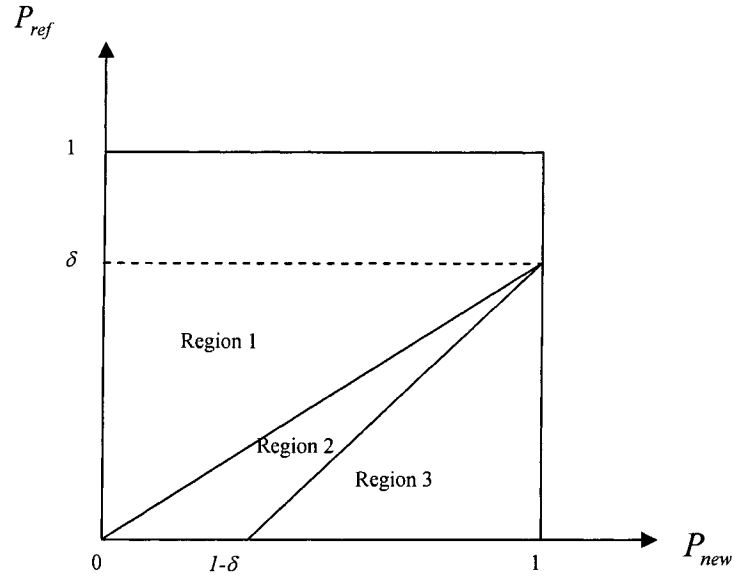
$\frac{P_{ref}}{\delta} = \frac{P_{new} - P_{ref}}{1 - \delta}$ , i.e.,  $P_{ref} = \delta P_{new}$ , there is no demand for refurbished products.

The proportion of consumers who will not buy any product is  $\frac{P_{ref}}{\delta}$ . The proportion of consumers with valuation less than  $\frac{P_{new} - P_{ref}}{1 - \delta}$  but greater than or equal to  $\frac{P_{ref}}{\delta}$  will buy refurbished products. The rest are willing to buy new products. The demands per unit time are scaled to represent the proportion of consumers in the market who are willing to buy products at the stated prices during the study period. The demand for new products is  $\lambda_{new} = 1 - \max\left(P_{new}, \frac{P_{new} - P_{ref}}{1 - \delta}\right)$  and the demand for refurbished products is  $\lambda_{ref} = \max\left(\frac{P_{new} - P_{ref}}{1 - \delta} - \frac{P_{ref}}{\delta}, 0\right)$ . The sum  $0 \leq \lambda_{new} + \lambda_{ref} \leq 1$  represents the proportion of all consumers per unit time who will buy this product. The relationship between demand, price and quality of refurbished products is illustrated in Figure 2. There are three different regions separated by the two linear equations  $P_{ref} = \delta P_{new}$  and  $P_{ref} = P_{new} - (1 - \delta)$ :

*Region 1:*  $P_{ref} \geq \delta P_{new}$ . There is no demand for refurbished products and the demand for new products equals  $\lambda_{new} = 1 - P_{new}$ .

*Region 2:*  $P_{new} - (1 - \delta) < P_{ref} < \delta P_{new}$ . There are demands for both new and refurbished products. We have  $\lambda_{new} = 1 - \frac{P_{new} - P_{ref}}{1 - \delta}$  and  $\lambda_{ref} = \frac{P_{new} - P_{ref}}{1 - \delta} - \frac{P_{ref}}{\delta}$ .

*Region 3:*  $P_{ref} \leq P_{new} - (1 - \delta)$ . There is no demand for new products and the demand for refurbished products equals  $\lambda_{ref} = 1 - \frac{P_{ref}}{\delta}$ .



**FIGURE 2. Ranger of prices with respect to refurbished product quality that govern demand for new and refurbished products.**

We will consider only cases in which the price of refurbished products will affect the total profit. When  $P_{ref}$  is less than  $P_{new} - (1 - \delta)$ , the demand for new products will be zero. As a result, even though the demand for refurbished products is higher, there will be no supply of products to refurbish and the manufacturer cannot make profit from both types of products. Similarly, when the value of  $P_{ref}$  is higher than  $\delta P_{new}$ , there is no demand for refurbished products so that the higher value of  $P_{ref}$  will not affect the total profit. Therefore, only the values of  $P_{ref}$  in the range  $[P_{new} - (1 - \delta), \delta P_{new}]$ , i.e., Region 2 and its boundary, will be considered. The other upper bound of  $P_{ref}$  is a condition from the steady state assumption of queueing networks as discussed in section 3.3 and Appendix 1.

### 3.2 The Queueing Network Model

The lowest tier of the supply chain including refurbishing is formulated as a queueing network model (Buzacott and Shantikumar, 1993). Similar approaches can be found in Toktay et al. (2000) and Souza et al. (2006). Figure 3 illustrates the whole system consisting of five stations plus arrivals. Each station represents a location or status of products.

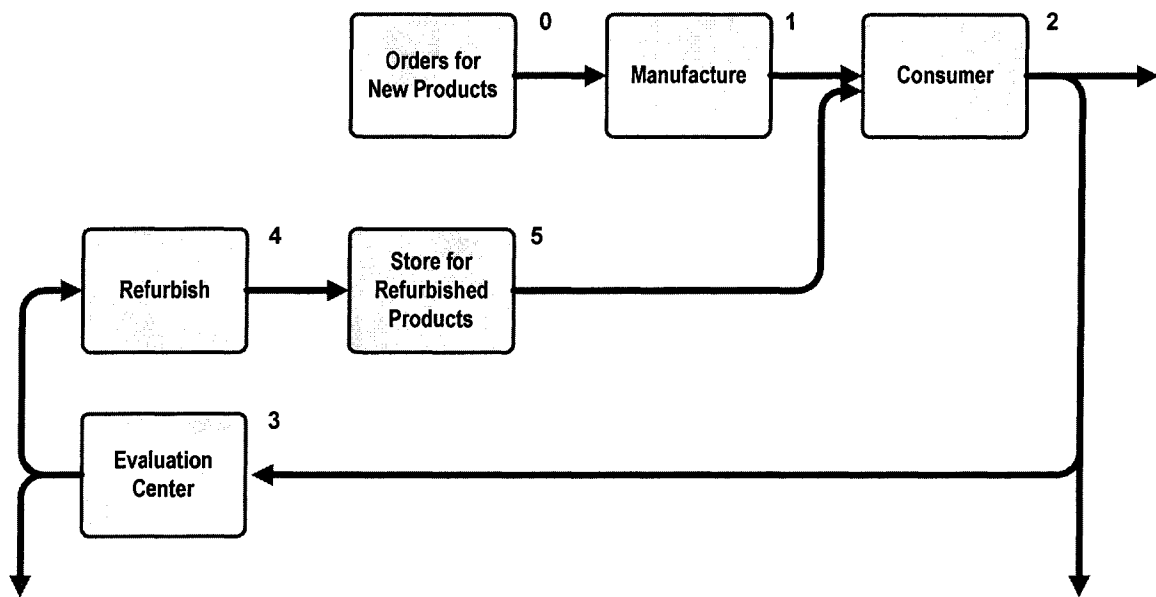


FIGURE 3. The open queueing network

According to the make-to-order policy, arrivals of the external demands for new products, at rate  $\lambda_{new}$ , cause materials to be released and authorize production to begin at the manufacturing site (station 1). New products are produced, and then distributed to consumers (station 2). A consumer receives and tries the product, and then decides whether to keep or to return it. The service time of station 2 represents the length of time the

consumer keeps the product before returning it; if the consumer chooses not to return the product, the product exits the system. The manufacturer evaluates all returned products at the evaluation site (station 3) and decides the proportions of products to be refurbished at the refurbishing site (station 4) or to be dismantled. The dismantling process is done outside the system so those products to be dismantled exit the system. Returned products are refurbished to a uniform quality and sold for the same price at station 5. At station 5, the service time interval represents the interarrival time between demands for refurbished products, or its residual in case of a product arrival to an empty store, and is assumed to be exponentially distributed. We assume no backorders for refurbished products; therefore, the demand arriving when there is no inventory will be considered lost. Station 0 is a virtual station that represents any activity that occurs outside the system. It serves as the origin of all jobs entering the system and the destination of all jobs exiting the system.

We assume that products are manufactured, evaluated, refurbished and sold one at a time. Therefore, all stations except station 2 are considered to have single servers. Station 2 is represented as an infinite-server station because the make-to-order policy guarantees that all new products produced will be used immediately by the consumers who order them. We use the word “job” as a general term to represent entities flowing in the queueing network. Jobs at station 1 are orders waiting to be processed. Jobs at stations 2, 3, 4 and 5 are finished products which can be new, returned or refurbished products. Jobs will be further classified into different types according to their routing and the number of times they have been returned. Let  $\eta$  be the maximum number of times that a product can be refurbished and resold to consumers. When  $\eta = 0$ , the system is the forward supply chain that consists only of stations 1 and 2. To be able to track the number of returns of the individual items, we

assume the manufacturer keeps records of this information from an automatic identification such as serial numbers tag or Radio Frequency Identification (RFID). Table 2 describes product types and their routings.

**TABLE 2. Product types and routings.**

<b>Job type</b>	<b>Description</b>	<b>Routing</b>
0	New products that are kept by the consumer after purchase.	{0,1,2,0}
1	Products that have been returned for the $l^{\text{th}}$ time from consumers and are sent to be dismantled	{2,3,0}
1'	Products that have been returned for the $l^{\text{th}}$ time from consumers that are sent to be refurbished for the $l^{\text{th}}$ time.	{3,4,5,2,0}

Jobs of type 0 or new products, after sold, will be either kept by the consumer or returned. If they are not returned, they will preserve their type; otherwise, their type will be changed at station 2 to type 1 or a first time returned product. Jobs of type 1 are evaluated and will preserve their type if they are sent to be dismantled. Jobs of type 1 will change at station 3 to type 1' or first time refurbished products if they are sent to be refurbished. After type 1' jobs have been sold, they will preserve their type if they are kept by the consumer and otherwise change at station 2 to type 2 if they are returned for the second time. When a job of type  $\eta'$  is returned, it becomes a type  $\eta + 1$  and is automatically sent to be dismantled. There is no job type  $(\eta + 1)'$ . Figure 4 shows the open queueing network for  $\eta = 1$ .



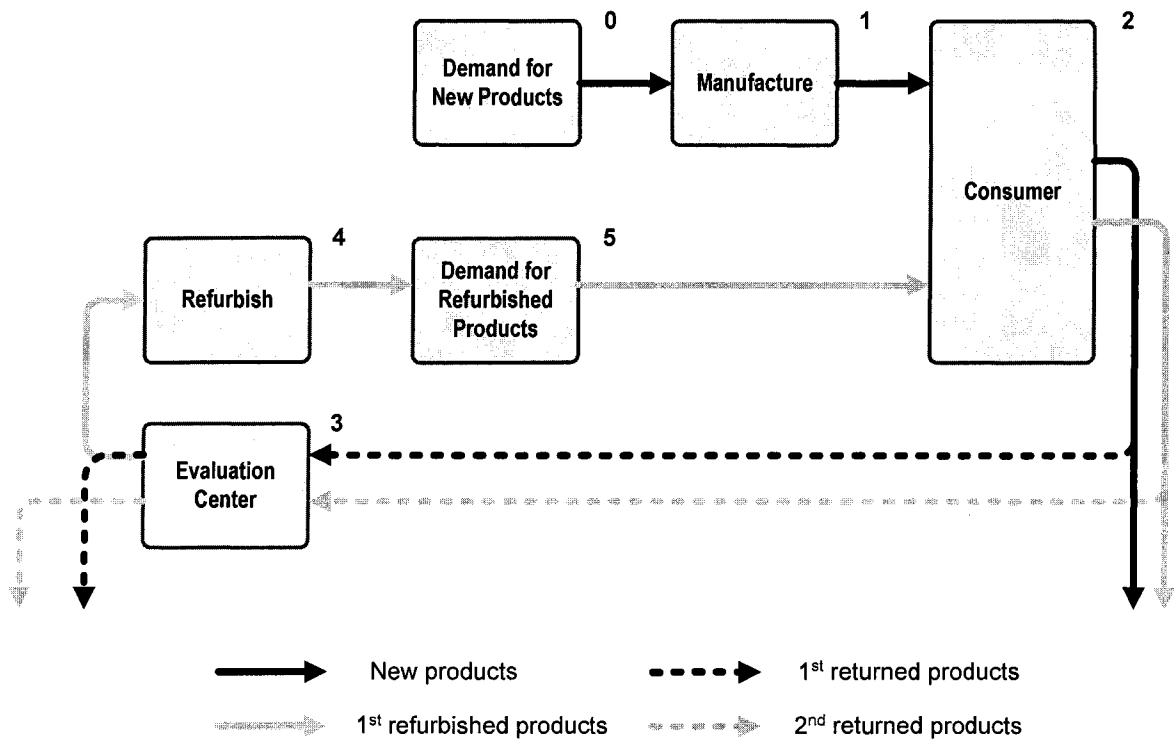


FIGURE 4. The open queueing network for  $\eta = 1$ .

The following subsections describe the queueing network model in detail. The first subsection is the open queueing network model for the general service time case, while the latter is the special case where the service time is exponentially distributed.

### 3.2.1 General Queueing Network Model

This is the general case of an open queueing network where the external interarrival time and the service times can have any distribution. There is still no exact solution for general networks; however we can use the two-moment approximation method to evaluate the performance of the system. We base our performance evaluation on the parametric-decomposition method for multiple-class  $GI/G/m$  open queueing networks developed similarly by Buzacott and Shantikumar (1993), Segal and Whitt (1989), Whitt (1994), and

Bitran and Morabito (1996). The comparison of the approximations with simulation can be found in Bitran and Tirupati (1988) and Whitt (1994).

The parametric-decomposition method evaluates each station  $i$  individually as a stochastically independent  $GI/G/m$  system by the set of parameters  $\{\lambda_i, c_{a_i}^2, E[S_i], c_{s_i}^2\}$ , where

$\lambda_i$  = expected arrival rate at station  $i$ ,

$c_{a_i}^2$  = squared coefficient of variation (scv) of the external interarrival time at station  $i$ ,

$E[S_i]$  = expected service time of aggregate jobs at station  $i$ ,

$c_{s_i}^2$  = scv of the service time of aggregate jobs at station  $i$ .

To obtain these parameters, several job types are treated as a single aggregate class under the assumptions of First Come First Served (FCFS) service protocol and unlimited waiting space (Whitt, 1983). Let

$A$  = set of arcs in the networks,  $A = \{01, 12, 20, 23, 30, 34, 45, 51\}$ .

For job type  $l, k = 0, 1, 1', \dots, \eta, \eta', \eta + 1$ ,

$\lambda_i^{(l)}$  = the expected arrival rate of job type  $l$  at station  $i$ ,

$\lambda_{0i}^{(l)}$  = the external arrival rate of job type  $l$  at station  $i$ ,

$\rho_i$  = the expected utilization of station  $i$ ,

$p_{ij}^{(k)(l)}$  = the probability that a job leaving station  $i$  as a type  $k$  job will join station  $j$  as a type  $l$  job,  $ij \in A$ ,

$p_{ij}$  = the transfer probability of aggregate jobs from station  $i$  to station  $j$ ,  $ij \in A$ ,

$p_{cr}$  = the probability that a product will be returned from the consumer to the manufacturer. We assume  $0 < p_{cr} < 1$  and it is independent of number of returns,

$p_{mr}^{(l)} = p_{mr}^{(l\bullet)}$  = the probability that the manufacturer will send an  $l^{th}$ -return of product (any grade) to be refurbished.

In our queueing network,  $p_{ij}^{(k)(l)}$  consists of  $p_{01}^{(0)(0)} = p_{12}^{(0)(0)} = 1$ ,  $p_{20}^{(0)(0)} = 1 - p_{cr}$ ,  $p_{23}^{(0)(1)} = p_{cr}$ . For  $l = 1, \dots, \eta$ ,  $p_{20}^{(l')(l')} = 1 - p_{cr}$ ,  $p_{23}^{(l')(l'+1)} = p_{cr}$ ,  $p_{30}^{(l)(l)} = 1 - p_{mr}^{(l)}$ ,  $p_{34}^{(l)(l')} = p_{mr}^{(l)}$  and  $p_{30}^{(\eta+1)(\eta+1)} = 1$ . The only external arrivals to the system occur at station 1, where the manufacturer receives orders for new products,  $\lambda_{01}^{(0)} = \lambda_{new}$ . From the traffic rate equations in

steady state,  $\lambda_i^{(l)} = \lambda_{0i}^{(l)} + \sum_{j=1}^5 \sum_k \lambda_j^{(k)} p_{ji}^{(k)(l)}$ ,  $i = 1, \dots, 5$ , we obtain

$$\lambda_1^{(0)} = \lambda_2^{(0)} = \lambda_{new},$$

$$\lambda_2^{(l')} = \lambda_4^{(l')} = \lambda_5^{(l')} = \lambda_{new} \prod_{k=1}^l p_{cr} p_{mr}^{(k)}, \quad l = 1, \dots, \eta,$$

$$\lambda_3^{(l)} = \lambda_{new} \prod_{k=1}^l p_{cr} p_{mr}^{(k-1)}, \quad l = 1, \dots, \eta + 1.$$

The arrival rates of aggregate jobs at station  $i$ ,  $\lambda_i = \sum_l \lambda_i^{(l)}$ , are  $\lambda_1 = \lambda_{new}$ ,  $\lambda_2 =$

$$\lambda_{new} \left( 1 + \sum_{l=1}^{\eta} \left( \prod_{k=1}^l p_{cr} p_{mr}^{(k)} \right) \right), \quad \lambda_3 = \lambda_{new} p_{cr} \left( 1 + \sum_{l=1}^{\eta} \prod_{k=1}^l p_{cr} p_{mr}^{(k)} \right) \quad \text{and} \quad \lambda_4 = \lambda_5 =$$

$\lambda_{new} \left( \sum_{l=1}^{\eta} \left( \prod_{k=1}^l p_{cr} p_{mr}^{(k)} \right) \right)$ . In addition, the aggregate arrival rate from station  $i$  to station  $j$  is

$\lambda_{ij} = \sum_l \lambda_{ij}^{(l)}$ , where  $\lambda_{ij}^{(l)} = \sum_k \lambda_i^{(k)} p_{ij}^{(k)(l)}$ ,  $i, j = 0, \dots, 5$ ,  $k, l = 0, 1, 1', \dots, \eta, \eta', \eta + 1$ . The transfer

probability of the aggregate jobs can be obtained by  $p_{ij} = \frac{\lambda_{ij}}{\lambda_i}$ ,  $i, j = 0, \dots, 5$ . Then, we obtain

the transition probability matrix of aggregate jobs as

$$\mathbf{P} = \begin{array}{c} \begin{matrix} & 0 & 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \left[ \begin{array}{cccccc} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 - p_{23} & 0 & 0 & p_{23} & 0 & 0 \\ 1 - p_{34} & 0 & 0 & 0 & p_{34} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{array} \right] \end{array}$$

In general, the FCFS policy well represents the processes of ordering, using, evaluating and refurbishing products in stations 1, 2, 3 and 5. However, at station 4 where returned products evaluated and arrived in different grades (Souza et al., 2002 and Guide et al., 2003), individual grades of returned products have different processing time and cost requirements to refurbish. Prioritizing products to be refurbished should increase the total profit. As a result, we implement the Shortest Expected Processing Time (SEPT) rule as an alternative to the FCFS service protocol at the refurbishing station. In the SEPT protocol, jobs with lower processing times have higher priority and will be processed before those with higher processing times. The previous studies by Wein and Ou (1991), and Buzacott and Shantikumar (1993) show that, when the arrivals follow a Poisson process and the processing time of each job is known, SEPT yields a lower cycle time of jobs than the FCFS service

protocol in many situations. Since our model is different from the previous studies and the aggregate arrivals do not follow a Poisson process, we are interested to see how this service protocol performs at station 4.

We use the approximation method proposed by Buzacott and Shantikumar (1993) that estimates the waiting time in SEPT from the waiting time in the FCFS service protocol. Therefore, at station 4 after jobs of different types and grades have been aggregated into grades only, all grades will be aggregated again into a single class to find the expected waiting time in FCFS. When  $\eta \geq 1$ , let

$\theta$  = number of grades of returned products,

$q^{(l,r)}$  = proportion of  $l^{\text{th}}$ -return products with grade  $r$ ,  $l = 1, \dots, \eta + 1$ ,  $r = 1, \dots, \theta$ ,

$$\sum_{r=1}^{\theta} q^{(l,r)} = 1,$$

$E[S_4^{(l,r)}]$  = the expected refurbishing time of  $l^{\text{th}}$ -return products with grade  $r$  at station 4,  $l = 1, \dots, \eta$ ,  $r = 1, \dots, \theta$ ,

$E[S_4^{(r)}]$  = the expected refurbishing time of products with grade  $r$  at station 4,  $r = 1, \dots, \theta$ ,

$\rho_4^{(l,r)}$  = the expected utilization of station 4 by  $l^{\text{th}}$ -return products with grade  $r$ ,  $l = 1, \dots, \eta$ ,  $r = 1, \dots, \theta$ ,

$p_{mr}^{(l,r)}$  = the proportion of  $l^{\text{th}}$ -return products with grade  $r$  to be refurbished,

$$\sum_{r=1}^{\theta} q^{(l,r)} p_{mr}^{(l,r)} = p_{mr}^{(l\bullet)} = p_{mr}^{(l)}, \quad l = 1, \dots, \eta \quad \text{and} \quad p_{mr}^{(0,r)} = 1, \quad r = 1, \dots, \theta,$$

$c_{s_4}^{2(\bullet r)}$  = the coefficient of variation of service time of aggregate products of grade  $r$  at station 4,  $r = 1, \dots, \theta$ .

Since the probability of sending jobs to station 4 can be differ according to type and grade, the aggregation procedure is not possible. Then we have multiple classes of jobs. Job type  $l$  and grade  $r$  at station 4 is now called job class  $(l, r)$ . The expected arrival rate, the service time and the scv of service time of aggregate products of grade  $r$  at station 4 are:

$$\begin{aligned}\lambda_4^{(\bullet r)} &= \lambda_{new} \sum_{l=1}^{\eta} p_{mr}^{(l,r)} q^{(l,r)} \prod_{k=1}^l p_{cr} \sum_{g=1}^{\theta} p_{mr}^{(k-1,g)} q^{(k-1,g)}, \text{ where } p_{mr}^{(0,g)} \equiv 1, g = 1, \dots, \theta. \\ E[S_4^{(\bullet r)}] &= \begin{cases} \frac{1}{\lambda_4^{(\bullet r)}} \sum_{l=1}^{\eta} \lambda_4^{(l,r)} E[S_4^{(l,r)}] & \text{if } \sum_l p_{mr}^{(l,r)} > 0, \\ 0 & \text{otherwise} \end{cases} \\ c_{s_4}^{2(\bullet r)} &= \begin{cases} \sum_{l=1}^{\eta} \frac{\lambda_4^{(l,r)}}{\lambda_4^{(\bullet r)}} \left( \frac{E[S_4^{(l,r)}]}{E[S_4^{(\bullet r)}]} \right)^2 (1 + c_{s_4}^{2(l,r)}) - 1 & \text{if } \sum_l p_{mr}^{(l,r)} > 0, \\ 0 & \text{otherwise.} \end{cases}\end{aligned}$$

Then, the aggregate parameters at station 4 are:

$$\begin{aligned}\lambda_4 &= \sum_{r=1}^{\theta} \lambda_4^{(\bullet r)} = \lambda_{new} \left( \sum_{l=1}^{\eta} \prod_{k=1}^l p_{cr} \sum_{g=1}^{\theta} p_{mr}^{(k,g)} q^{(k,g)} \right) \\ E[S_4] &= \frac{1}{\lambda_4} \sum_{r=1}^{\theta} \lambda_4^{(\bullet r)} E[S_4^{(\bullet r)}], \text{ and} \\ c_{s_4}^2 &= \sum_{r=1}^{\theta} \frac{\lambda_4^{(\bullet r)}}{\sum_{r=1}^{\theta} \lambda_4^{(\bullet r)}} \left( \frac{E[S_4^{(\bullet r)}]}{E[S_4]} \right)^2 (1 + c_{s_4}^{2(\bullet r)}) - 1.\end{aligned}$$

After we obtain the parameters and probability matrix of aggregate jobs, we now find the approximation of the scv of interarrival times at station  $i$  ( $c_{a_i}^2$ ). Assuming that the scv of

the service time of aggregate jobs at station  $i$ ,  $c_{s_i}^2$ ,  $i = 1, \dots, 5$ , are known,  $c_{a_i}^2$  can be found by solving the system of equations derived from three basic processes (Whitt, 1983): (i) merging or superposition of arrivals, (ii) departure, and (iii) decomposition or splitting of departures.

(i) *Superposition of arrivals*. This method is a hybrid approximation of two methods: the asymptotic method and the stationary-interval method. The scv of interarrival times is approximated by

$$c_{a_i}^2 = w_i \sum_{j=0}^5 \frac{\lambda_{ji}}{\lambda_i} c_{a_{ji}}^2 + 1 - w_i$$

where

$$w_i = \frac{1}{1 + 4(1 - \rho_i)^2 (v_i - 1)}$$

$$v_i = \frac{1}{\sum_{j=0}^5 \left( \frac{\lambda_{ji}}{\lambda_i} \right)^2}$$

$$\rho_i = \frac{\lambda_i E[S_i]}{m_i}, i, j = 1, \dots, 5, \text{ where } m_i \text{ is number of servers at station } i.$$

Note that in our model  $m_1 = m_3 = m_4 = m_5 = 1$  and  $m_2 = \infty$ . Therefore  $\rho_i = \lambda_i E[S_i]$  for  $i = 1, 3, 4, 5$  and  $\rho_2 = 0$ .

(ii) *Departure*. This process finds the overall scv of the time between departures at station  $i$  ( $c_{d_i}^2$ ) as

$$c_{d_i}^2 = 1 + (1 - \rho_i^2)(c_{a_i}^2 - 1) + \frac{\rho_i^2(c_{s_i}^2 - 1)}{\sqrt{m_i}}, i = 1, \dots, 5.$$

(iii) *Splitting*. This process finds the scv of departures or arrivals of jobs from station  $i$  to station  $j$ ,  $c_{d_{ij}}^2$ . There are two different approximations of  $c_{d_{ij}}^2$ , one for deterministic routing and the other one for probabilistic routing. Because routings in our network are both deterministic, e.g.,  $\{0,1,2\}$ ,  $\{4,5,2\}$  and probabilistic ( $\{2,3\}$ ,  $\{2,0\}$ ,  $\{3,4\}$ ,  $\{3,0\}$ ), we use the hybrid combination of both approximations proposed by Segal and Whitt (1989) to capture the overall effect.

For probabilistic routing (Whitt, 1983), the scv of departures or arrivals of aggregate jobs that move from station  $i$  to station  $j$  with probability  $\hat{p}_{ij}$  is  $c_{d_{ij}}^2 = c_{a_{ij}}^2 = \hat{p}_{ij}c_{d_i}^2 + 1 - \hat{p}_{ij}$ .

On the other hand, for deterministic routing, Whitt (1994) proposes the scv of departures of type  $l$  at station  $i$ ,  $c_{d_i}^{2(l)} = d_i^{(l)}c_{d_i}^2 + (1 - d_i^{(l)})d_i^{(l)}\bar{c}_{a_i}^{2(l)} + (1 - d_i^{(l)})^2c_{a_i}^{2(l)}$  where  $d_i^{(l)}$  is the proportion

of all departures at station  $i$  that are type  $l$ ,  $c_{a_i}^{2(l)}$  = the scv of the arrival of type  $l$  and  $\bar{c}_{a_i}^{2(l)}$  =

the scv of the arrival of all types except type  $l$ ,  $\bar{c}_{a_i}^{2(l)} = \frac{c_{a_i}^2 - \frac{\lambda_i^{(l)}}{\lambda_i}c_{a_i}^{2(l)}}{\left(1 - \frac{\lambda_i^{(l)}}{\lambda_i}\right)}$ .

As discussed above, all jobs types that flow from station  $i$  to station  $j$  are treated similarly, therefore, we simplify the process by assuming  $c_{a_i}^2 \approx c_{a_i}^{2(l)}$ . Then the above

equation can be rewritten in form of  $c_{d_{ij}}^2$  of aggregate jobs such that  $c_{d_{ij}}^2 = \hat{d}_{ij}c_{d_i}^2 + (1 - \hat{d}_{ij})c_{a_i}^2$

where  $\hat{d}_{ij}$  is the transfer probability of aggregate jobs from  $i$  to  $j$  of deterministic routing. In our case  $\hat{d}_{ij} = 1$  for  $ij = \{01, 12, 45, 52\}$ .

The hybrid approximation is the convex combination of both  $c_{d_{ij}}^2$  in probabilistic and

deterministic routing,  $c_{d_{ij}}^2 = (1 - \beta_{ij})(\hat{p}_{ij}c_{d_i}^2 + 1 - \hat{p}_{ij}) + \beta_{ij}(\hat{d}_{ij}c_{d_i}^2 + (1 - \hat{d}_{ij})c_{a_i}^2)$ , where



$\hat{d}_{ij} \in \{p_{01}, p_{12}, p_{45}, p_{52}\}$ ,  $\hat{p}_{ij} \in \{p_{20}, p_{23}, p_{30}, p_{34}\}$  and  $\beta_{ij}$  is the proportion of all flow from  $i$  to  $j$  that is due to deterministic routing (as opposed to probabilistic routing) (Segal and Whitt, 1989).

The expected waiting time of aggregate jobs in each station can be approximated by the parameters  $\{\lambda_i, c_{a_i}^2, \mu_i, c_{s_i}^2\}$  obtained from the previous step:

$$E[W_i]_{GI/G/m_i:FCFS} \approx \frac{(c_{a_i}^2 + c_{s_i}^2)g_i}{2} E[W_i]_{M/M/m_i:FCFS}, \quad i = 1, \dots, 5$$

where

$$g_i = \begin{cases} \exp\left[-\frac{2(1-\rho_i)(1-c_{a_i}^2)^2}{3\rho_i(c_{a_i}^2 + c_{s_i}^2)}\right], & c_{a_i}^2 < 1 \\ 1, & c_{a_i}^2 \geq 1 \end{cases}$$

$$E[W_i]_{M/M/m_i:FCFS} = \frac{(m_i\rho_i)^{m_i} \pi_i(0)}{\mu_i m_i (1-\rho_i)^2 m_i!},$$

$$\pi_i(0) = \left\{ \sum_{j=0}^{m_i-1} \frac{(m_i\rho_i)^j}{j!} + \frac{(m_i\rho_i)^{m_i}}{(1-\rho_i)m_i!} \right\}^{-1}.$$

The expected number of jobs at station  $i$  for FCFS service protocol is

$$E[N_i]_{GI/G/m_i:FCFS} = \rho_i + \lambda_i E[W_i]_{GI/G/m_i:FCFS},$$

$$E[N_2]_{GI/G/m_i:FCFS} = \lambda_2 E[S_2].$$

At station 4, the expected waiting time for job class  $(l, r)$ , is,

$$E[W_4^{(l,r)}]_{GI/G/1:SEPT} \approx \frac{1-\rho_4}{(1-\rho_4^{(l,r)})(1-\rho_4^{(k,s)})} E[W_4]_{GI/G/1:FCFS}, \quad \text{where } (k, s) \text{ is the job class}$$

with next lower priority to jobs  $(l, r)$ , and

$$\rho_4^{(l,r)} = \lambda_4^{(l,r)} E[S_4^{(l,r)}].$$

The expected number of aggregate jobs at station 4 is

$$E[N_4]_{GI/G/1:SEPT} = \rho_4 + \sum_{l,r} \left( \frac{\lambda_4^{(l,r)}(1-\rho_4)}{(1-\rho_4^{(l,r)})(1-\rho_4^{(k,s)})} \right) E[W_4]_{GI/G/1:FCFS}.$$

### 3.2.2 Jackson Open Queueing Network Model

This section presents the special case of the previous model where each station has an exponentially distributed service time and serves jobs according to the FCFS service protocol. Products of different grades and types are aggregated into a single class. The returned products can be refurbished indefinitely, i.e.,  $\eta = \infty$ . The only decision variables are  $P_{ref}$  and  $p_{mr}$ , which represents the proportion of aggregate returned products to be refurbished. This network is a single-class Jackson queueing network. The performance of each station can be evaluated independently (see Buzacott and Shantikumar, 1993 p.315). In this queueing network model,  $p_{23} = p_{cr}$  and  $p_{34} = p_{mr}$ . The routing matrix for aggregate jobs is

$$\mathbf{P} = \begin{matrix} & \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1-p_{cr} & 0 & 0 & p_{cr} & 0 & 0 \\ 1-p_{mr} & 0 & 0 & 0 & p_{mr} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

The arrival rate at each station can be computed by the traffic equations,  $\lambda_i =$

$$\lambda_0 p_{0i} + \sum_{j=1}^n \lambda_j p_{ji}, i = 1, \dots, 5. \text{ We have } \lambda_1 = \lambda_{new}, \lambda_2 = \frac{\lambda_{new}}{1 - p_{cr} p_{mr}}, \lambda_3 = \frac{p_{cr} \lambda_{new}}{1 - p_{cr} p_{mr}}, \lambda_4 =$$

$$\lambda_5 = \frac{p_{cr} p_{mr} \lambda_{new}}{1 - p_{cr} p_{mr}}. \text{ The expected number of jobs at station } i, i = 1, 3, 4, 5 \text{ is } E[N_i]_{M/M/1:FCFS}$$

$$= \frac{\rho_i}{1 - \rho_i}. \text{ The expected number of products with consumers before being returned is}$$

$$E[N_2]_{M/M/\infty:FCFS} = \lambda_2 E[S_2].$$

We can view this model as a simplified version of the general case. Nevertheless, the model yields the exact result as opposed to approximation in the general case and it is smaller so that it requires significantly less time to solve. We use this model to test different parameters and compare the results to the model with general service times in Chapter 4.

### 3.3 The Model Formulation and its Optimality Conditions

Holding and transferring cost may depend on the type and grade of products. In this research we assume that holding costs are uniform over all types and grades while the costs of transferring jobs to all stations are uniform over types but not over grades. Since jobs transferring to all stations except station 4 are aggregated into single grade, we introduce the following notations to compute the total profit.

$$h_i = \text{cost per unit time per item being held in station } i, i = 1, \dots, 5,$$

$$c_{ij} = \text{scaled cost of transferring a job from station } i \text{ to station } j, ij \in A \setminus \{34\},$$

$$c_{34}^{(r)} = \text{scaled cost of transferring a job of grade } r \text{ from station 3 to station 4.}$$

In particular, the cost of transferring a job from station  $i$  to station  $j$  may consist of handling, production and/or transportation cost. Table 3 shows the breakdown of  $c_{ij}$ .

TABLE 3. Breakdown of costs in  $c_{ij}$ .

	Handling	Production	Transportation
$c_{01}$	×	×	×
$c_{12}$	×		×
$c_{20}$	×		
$c_{23}$	×		×
$c_{30}$	×		×
$c_{34}$	×		×
$c_{45}$	×	×	
$c_{52}$	×		×

The optimality conditions of the general queueing and the Jackson open queueing network models are different. We start with the general case then follow with the Jackson open queueing model.

### 3.3.1 General Queueing Network Model

For the general queueing network model, jobs transferring from station 3 to 4 are classified into types  $l$  and grades  $r$ . At this point, costs of holding and transferring jobs are assigned according to their type and grades. For  $r = 1, \dots, \theta$  and  $l = 1, \dots, \eta$ ,

$c_{34}^{(l,r)}$  = cost of refurbishing a returned products of class  $(l,r)$ ,

$P_{dis}^{(l,r)}$  = price for a dismantled product of class  $(l,r)$ .

The formula for the total profit is

Total Profit = Revenue New + Revenue Refurbish + Revenue Dismantle

Transferring Cost + Inventory Cost

$$\text{Revenue New} = P_{new} \lambda_{new} (1 - p_{cr})$$

$$\text{Revenue Refurbish} = P_{ref} \lambda_4 (1 - p_{cr})$$

$$\begin{aligned} \text{Revenue Dismantle} &= \lambda_{new} \sum_{r=1}^{\theta} \sum_{l=1}^{\eta} P_{dis}^{(l,r)} q^{(l,r)} (1 - p_{mr}^{(l,r)}) \prod_{k=1}^l p_{cr} \sum_{g=1}^{\theta} p_{mr}^{(k-1,g)} q^{(k-1,g)} + \\ &\lambda_{new} \sum_{r=1}^{\theta} P_{dis}^{(\eta+1,r)} q^{(\eta+1,r)} \prod_{k=1}^{\eta+1} p_{cr} \sum_{g=1}^{\theta} p_{mr}^{(k-1,g)} q^{(k-1,g)} \end{aligned}$$

$$\text{Transferring Cost} = \sum_{ij \in A | ij \neq \{34\}} c_{ij} \lambda_{ij} + \lambda_{new} \sum_{r=1}^{\theta} \sum_{l=1}^{\eta} c_{34}^{(l,r)} q^{(l,r)} p_{mr}^{(l,r)} \prod_{k=1}^l p_{cr} \sum_{g=1}^{\theta} p_{mr}^{(k-1,g)} q^{(k-1,g)}$$

$$\text{Inventory Cost} = \sum_{i=1}^5 h_i E[N_i]$$

At station 4, returned products that require less refurbishing time will incur less inventory cost. Therefore, to maximize total profit, products that require less refurbishing cost and time should be sent to be refurbished first. Realistically, this process occurs at the evaluation center where returned products are sorted according to their grade. Products with better grade require less cost and time to refurbish. Then the priority can be assigned according to grades; the higher the grade, the higher the priority. On the other hand, the type of products or the number of times products have been returned does not obviously affect the time and cost to refurbish as much as their grades. Nevertheless, the higher number of returns implies the longer time the product has been circulated in the market and the possibility of a higher amount of usage. We experiment with the effect of the maximum

number of returns and prioritizing products according to grade and type compared with other policies in Chapter 4.

When there is no priority, the decision variables consist only of  $P_{ref}$  and  $p_{mr}$ . Returned products with any grade and return history will be sent to be refurbished with probability  $p_{mr}$ . Therefore  $p_{mr}^{(l,r)} = p_{mr}$ ,  $l = 1, \dots, \eta$ ,  $r = 1, \dots, \theta$ . For the model with grade priority only, decision variables consist of  $P_{ref}$  and  $p_{mr}^{(\bullet r)}$ ,  $r = 1, \dots, \theta$ . In this case, every time a product is returned in grade  $r$ , a proportion  $p_{mr}^{(\bullet r)}$  of them will be refurbished, i.e.,  $p_{mr}^{(l,r)} = p_{mr}^{(\bullet r)}$  for each  $l$ . When priority of returned products is assigned based on both grade and number of returns, the variables consist of  $P_{ref}$  and  $p_{mr}^{(l,r)}$ ,  $l = 1, \dots, \eta$ ,  $r = 1, \dots, \theta$ . A higher number of decision variables introduces more complexity to the optimization model, but as shown below, we can reduce the complexity of the problem by optimizing only  $P_{ref}$  and a single selected  $p_{mr}^{(\bullet r)}$  or  $p_{mr}^{(l,r)}$  at a time.

For grade and number of returns priority, the problem is broken down to  $l \times r$  sub-problems. Let product classes  $\mathbf{H}$  be the set of job classes with higher priority than  $(l, r)$  and  $\mathbf{L}$  be the set of job classes with lower priority than  $(l, r)$ . Then  $p_{mr}^{(k,s)} = 1$ ,  $\forall (k, s) \in \mathbf{H}$  and  $p_{mr}^{(m,t)} = 0$ ,  $\forall (m, t) \in \mathbf{L}$ . The job class  $(l, r)$  of interest will be the decision variable and have the value in the interval  $[0, 1]$ , i.e.,  $0 \leq p_{mr}^{(l,r)} \leq 1$ , while jobs in other classes will have value either zero or one. For example, let  $\eta = 2$ ,  $\theta = 3$  and  $\succ$  mean ‘‘has higher priority than’’. Products with different types and grades are sorted according to their time and cost to refurbish as follows:  $(l, r) = (1, 1) \succ (2, 1) \succ (1, 2) \succ (2, 2) \succ (1, 3) \succ (2, 3)$ . When  $p_{mr}^{(2,2)}$  is

a decision variable, then  $0 \leq p_{mr}^{(2,2)} \leq 1$ .  $H = \{(1,1), (2,1), (1,2)\}$  and  $L = \{(1,3), (2,3)\}$ . We now have six sub-problems. The (sub) optimal solution from each of these sub-problems will be compared to find the optimal solution that maximizes the total profit.

Let  $f(P_{ref}, p_{mr}^{(l,r)})$  be the total profit function in terms of the decision variables  $P_{ref}$  and  $p_{mr}^{(l,r)}$ ,  $l = 1, \dots, \eta$  and  $r = 1, \dots, \theta$ . The optimization model is

$$\text{Max } f(P_{ref}, p_{mr}^{(l,r)})$$

**Subject to:**

$$P_{ref} \geq P_{new} - (1 - \delta) \quad (1)$$

$$P_{ref} \leq \delta P_{new} \quad (2)$$

$$\rho_1 \leq 1 - \varepsilon_1 \quad (3)$$

$$\rho_3 \leq 1 - \varepsilon_3 \quad (4)$$

$$\rho_4 \leq 1 - \varepsilon_4 \quad (5)$$

$$\rho_5 \leq 1 - \varepsilon_5 \quad (6)$$

$$\rho_5 \geq \gamma \quad (7)$$

$$p_{mr}^{(l,r)} \leq 1 \quad (8)$$

$$p_{mr}^{(l,r)} \geq 0 \quad (9)$$

where  $\varepsilon_i$ ,  $i = 1, 3, 4, 5$ , are some small positive constants. Constraints (1) and (2) come from the demand assumptions in Section 3.1 to guarantee the nonnegativity of demand for new and refurbished products. Inequalities (3) to (6) are the nonstrict inequality form of steady-state conditions, suitable for optimization software. At station 5 where the demand for

refurbished products occurs, constraint (7) assigns a minimum service level,  $\gamma$ , for the refurbished products store. The service level is typically known as one minus the ratio of lost sales to the maximum possible sales. In our model, the maximum possible sales equals  $\lambda_{ref}$  and actual total sales rate of refurbished products is  $\lambda_5$ . Therefore the service level is  $1 - \frac{\lambda_{ref} - \lambda_5}{\lambda_{ref}}$  or one minus the utilization of station 5 which is the ratio of the supply rate to the demand rate of refurbished products,  $\rho_5 = \lambda_5 E[S_5] = \frac{\lambda_5}{\lambda_{ref}}$ . We set  $\gamma > 0$  to assure that when the manufacturer sets the price of refurbished products to the point that their demand occurs, there are some refurbished products to supply these demands. More details about  $\gamma$  can be found in case 2 of Appendix 1. Constraints (8) and (9) are bounds on the probability  $p_{mr}^{(l,r)}$ . Constraints (5) and (6) are immaterial when  $p_{mr}^{(l,r)} = 0$  because there are no arrivals to stations 4 or 5.

In line with Souza et al. (2002), we assume the time to evaluate returned products is short enough that the manufacturer should have ample capacity to handle all returned products; therefore, we will discard constraint (4). We also assume that the refurbishing site has enough capacity to process the maximum possible rate of incoming returned products,

i.e.,  $\mu_4 > \lim_{p_{mr}^{(k)} \rightarrow 1} \lambda_{new} \left( \sum_{l=1}^{\eta} \left( \prod_{k=1}^l p_{cr} p_{mr}^{(k)} \right) \right)$ , which implies that (5) is less restrictive than (2). It

can be shown in turn that (6) renders (2) redundant while (7) is more restrictive than (1). Therefore the feasible region is defined by (3), (6), (7), (8) and (9). The feasible region is convex if products can be refurbished and returned infinitely. On the other hand, we prove in

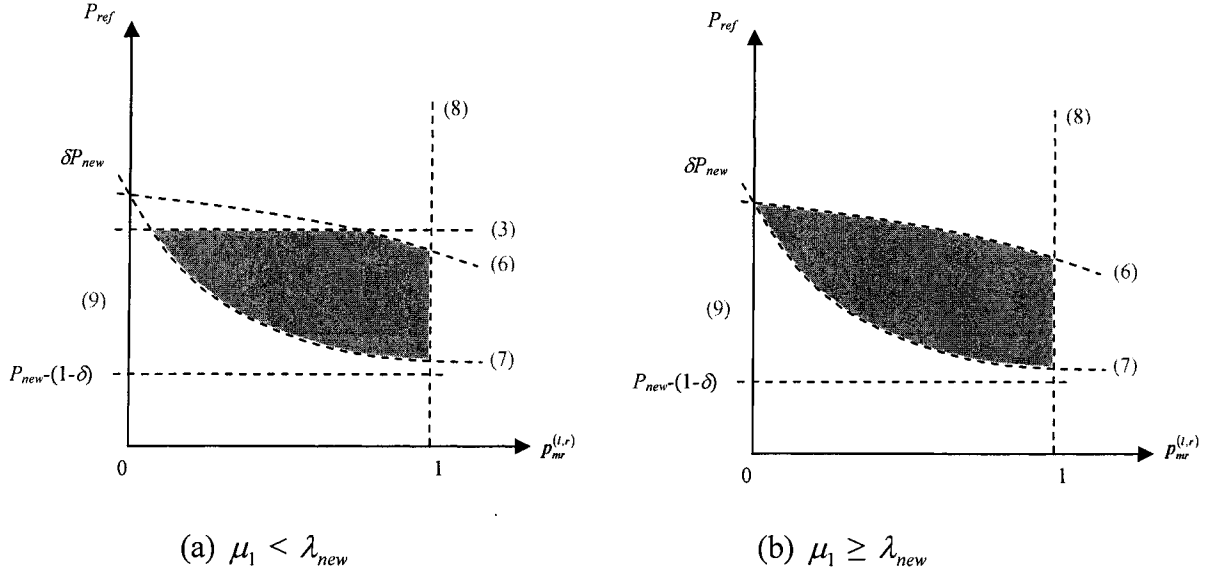


Appendix 1 that the feasible region is convex when  $\eta \rightarrow \infty$ ; but it is not convex when the maximum number of times products can be refurbished is limited.

Because the total profit is not necessarily pseudoconcave over the entire feasible region, there may be multiple local optima. Although these local optimal solutions could lie on any point in the feasible region, a closer examination of the objective function shows that points on boundary of (3) and (6) do not yield the maximum total profit because the high inventory cost outweighs the revenue. Therefore (3) and (6) will not be active at an optimal solution. From the Karush-Kuhn-Tucker (KKT) optimality conditions, we find three possibilities for locally optimal solutions:

- 1) The solution lies on the point  $(P_{ref}, p_{mr}^{(l,r)}) = (\delta P_{new}, 0)$  when new product manufacturing capacity exceeds demand for new products (Figure 5b), or at the point where (3) and (7) are both binding constraints when the manufacturing capacity is less than the demand for new products (Figure 5a).
- 2) The solution is in the interior of the feasible region, i.e.,  $0 < p_{mr}^{(l,r)} < 1$  and  $P_{ref}$  lies below constraint (3) or (6) and above (7).
- 3) The optimal solution lies on the boundary  $p_{mr}^{(l,r)} = 1$  and  $P_{ref}$  lies below constraint (3) or (6) and above (7).

The details of the KKT conditions can be found in Appendix 2.



**FIGURE 5. Two feasible regions depending on the relationship between demand and manufacturing capacity for new products.**

Points corresponding to case (1) can be identified simply, and correspond to not refurbishing product class  $(l,r)$ , or refurbishing the minimum amount required to relieve the manufacturing capacity constraint. When the feasible region is convex points corresponding to case (2), can be found by a hill-climbing procedure from an initial value of  $p_{mr}^{(l,r)}$  between 0 and 1. When the feasible region is not convex, we believe that the optimization algorithm in Mathematica 5.0 can correctly find the local optimum. We verified this by comparing the optimal total profit found by Mathematica with other total profits in an  $\varepsilon$ -neighborhood around the stopping point. Furthermore, the plot of total profit vs the decision variables shows only two possible saddle points in the interior of feasible region, one local minimum and one local maximum. The local optimum found by Mathematica is the local maximum one. The third case is simply optimizing  $P_{ref}$  at the point  $p_{mr}^{(l,r)} = 1$ . Then we can find the

global optimum for each  $(l, r)$  by comparing profits for the locally optimal points from cases 1, 2 and 3. The maximum total profit and corresponding optimal solution for all possible value of  $P_{ref}$  and  $p_{mr}^{(l,r)}$  can be found by comparing optimal total profits from all  $(l, r)$ . In the following section, we will see sensitivity analysis and comparative statics from numerical results and discuss the conditions under which the different cases are optimal.

### 3.3.2 Jackson Open Queueing Network Model

Since returned products have only one aggregate type and grade, the total profit function consists of decision variables  $P_{ref}$  and  $p_{mr}$ .

$$\begin{aligned} \text{Total Profit} &= \text{Revenue New} + \text{Revenue Refurbish} + \text{Revenue Dismantle} - \\ &\quad \text{Transferring Cost} + \text{Holding Cost} \end{aligned}$$

$$\text{Revenue New} = P_{new} \lambda_{new} (1 - p_{cr})$$

$$\text{Revenue Refurbish} = P_{ref} \lambda_5 (1 - p_{cr})$$

$$\text{Revenue Dismantle} = P_{dis} \lambda_3 (1 - p_{mr})$$

$$\text{Transferring cost} = \sum_j \sum_i c_{ij} \lambda_i p_{ij}$$

$$\text{Holding cost} = \sum_i h_i E[N_i]$$

Similar to the general case, let  $f(P_{ref}, p_{mr})$  be the function for total profit. The optimization model is

$$\text{Max } f(P_{ref}, p_{mr})$$

**Subject to:**

Constraints (1) to (7) from the general case and

$$p_{mr} \leq 1 \quad (10)$$

$$p_{mr} \geq 0 \quad (11)$$

where  $\varepsilon_i$ ,  $i = 1, 3, 4, 5$ , are some small positive constants. The KKT conditions are also similar to the general case:

- 1) The solution lies on the minimal point  $(P_{ref}, p_{mr}) = (\delta P_{new}, 0)$  when new product manufacturing capacity exceeds demand, or where (3) and (7) are both binding when it does not.
- 2) The solution is in the interior of the feasible region, i.e.,  $0 < p_{mr} < 1$  and  $P_{ref}$  lies below constraint (3) or (6) and above (7).
- 3) The optimal solution lies on the boundary  $p_{mr} = 1$  and  $P_{ref}$  lies below constraint (3) or (6) and above (7).

## CHAPTER 4. NUMERICAL EXAMPLES

We perform numerical experiments to discover how the optimal price and quantity of refurbished products are affected by the following parameters: 1) *Cost and quality of refurbished products*. From Chapter 1, we see that each company has its own policy for setting the perceived quality of refurbished products. The quality of refurbished products is studied along with the cost of refurbishing products that directly affects the quality. 2) *Price of new products and their backorder cost*. New product sales account for the largest portion of revenue. The price of new products affects the relationship between the demand and manufacturing capacity which may result in high backorder cost. 3) *Variability of service time at stations*. The variability of service time affects inventory cost. We assign the appropriate variability of service times to stations in the general network, then compare the optimal objective function and decision variable values with the special case of exponential service times. 4) *The refurbishing policy*. The policy includes the possibility of evaluating and sorting returned products, and prioritizing products for refurbishing by their grades and return history.

### 4.1 Jackson Open Queueing Network Model

We designed a numerical study to explore the demand and cost characteristics of new and refurbished products that would encourage or discourage refurbishing and influence the price of refurbished products. Specifically, we study the impact of  $P_{new}$ , the backorder penalty for new products  $h_1$ , the perceived quality  $\delta$ , and the cost of refurbishing  $c_{45}$ . Other

parameter values were set to represent a real situation as closely as possible. The probability that a consumer would return a product was  $p_{cr} = 0.25$ , based on the 15 – 20% rate of commercial returns for high-tech products (Toktay et al., 2003) plus additional returns from leasing and other sources. Given that the prices are normalized between 0 and 1, we set  $c_{12}$ , the manufacturing variable cost, to 0.25 so that  $P_{new} \geq 0.3$  would provide a reasonable profit margin. Other costs were set in relation to  $c_{12}$ , as  $c_{23} = 0$ ,  $c_{20} = 0$ ,  $c_{34} = 0.01$ ,  $c_{30} = 0.02$ , and  $c_{52} = 0$ . Holding costs were  $h_2 = 0$  and  $h_i = 0.00005$ ,  $i = 3, \dots, 5$ . These holding costs per unit time appear small because they are scaled twice, first by a price factor and second by a time factor; for instance, an item that cost \$500 to produce at the rate of 300 per unit time would have a holding cost of  $(500/0.25)(300/0.6)h_i = \$50$  per month. Other combinations of cost and production rate would scale holding costs differently. The selling price for components of a dismantled product was  $P_{dis} = 0.15$ .

Given a minimum value of 0.3 for  $P_{new}$ , the demand rate  $\lambda_{new} \leq 0.7$ . The manufacturing rate  $\mu_1$  was set to 0.6, so that demand would be less than capacity at station 1 in most but not all cases. Correspondingly  $\mu_2$ ,  $\mu_3$  and  $\mu_4$  were set to 0.006 and 0.6 and 0.3, respectively. The consumer evaluation rate  $\mu_2$  is much lower than rates at other stations because the mean time products are held by consumers is very long compared with the time to manufacture them. Nevertheless, the consumer station has unlimited capacity due to its infinite number of servers. As in Souza et al. (2002), the mean evaluation time should be quite short; however, we set  $\mu_3 = 0.6$  to reflect the fact that resources such as manpower may not be continuously available and to provide a nontrivial utilization for station 3. We also set

the refurbishing rate  $\mu_4$  low enough for its utilization to be noticeable but not high enough for its utilization to constrain the optimal solution.

Taken together, the parameter values allowed examination of the tradeoffs between profits to be made from new and refurbished products and potential problems of long waits for new products or high inventories of refurbished products. The numerical example was solved by Mathematica (Wolfram Research, Inc., 2003) and LINGO (Lindo, Inc., 2004) software.

Figure 6 illustrates the three cases of global optimum by plotting the optimal profit as a function of  $p_{mr}$  when  $P_{new}$  is set to 0.45 so that there no capacity constraint for manufacturing new products. There are at most two local optima at each value of  $\delta$ . As suggested by case 1 of the KKT conditions,  $\partial f / \partial p_{mr} < 0$  for small values of  $p_{mr}$ , so that  $(P_{ref}, p_{mr}) = (\delta P_{new}, 0)$  is a local optimum in each case. Therefore, a case 2 local optimum in the interior of the feasible region is separated from the case 1 solution by a significant margin, suggesting that it is never optimal to refurbish only a small fraction of returns. By a careful examination of the different components of profit, we observe that a small increase from  $p_{mr} = 0$  has two negative effects on profit: (1) the demand for new products falls, causing all three types of revenue to decrease because  $\lambda_3$  and  $\lambda_5$  decrease in proportion to  $\lambda_{new}$ , and (2) the inventory cost at station 5 rises sharply because  $P_{ref}^*(p_{mr})$  close to  $\delta P_{new}$  creates little demand for refurbished items. For larger values of  $p_{mr}$  (with decreasing  $P_{ref}$ ), the slope becomes positive as the rate of increase in total profit per new item produced exceeds the rate of decrease in  $\lambda_{new}$ . When the perceived quality is low, the slope is negative

at high  $p_{mr}$  because at the correspondingly low values of  $P_{ref}$  it is more profitable to dismantle some items. Note that the optimal policy has this discontinuous character even without fixed costs for setting up the refurbishment processes nor economies of scale. Table 4 shows the comparisons to identify the global optimum in each case.

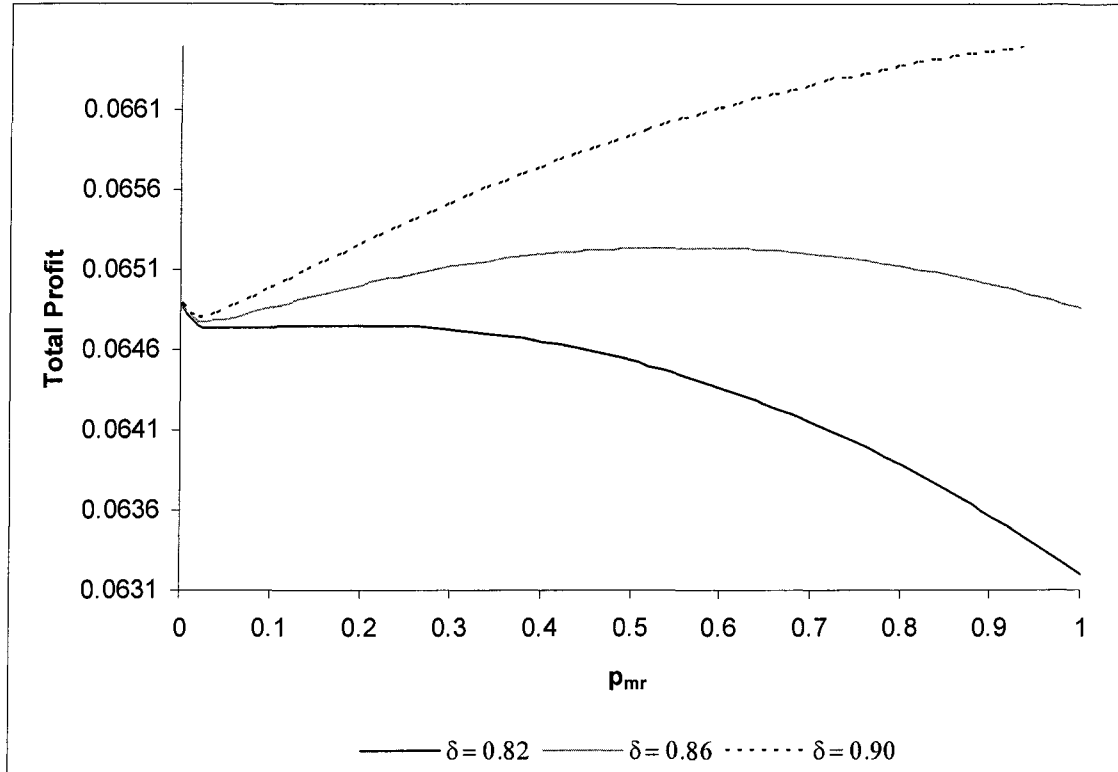


FIGURE 6 Total profit for different values of  $p_{mr}$  for  $P_{new} = 0.45$ ,  $c_{45} = 0.06$ ,  $h_1 = 0.0001$  and  $\delta = 0.82, 0.86$  and  $0.90$

TABLE 4. Comparing local optima to find the global optimum.

$\delta$	Local optima $(P_{ref}, p_{mr})$
0.82	<b>(0.3690,0)</b> , (0.3648,0.18)
0.84	(0.3870,0), <b>(0.3769,0.56)</b>
0.90	(0.4050,0), <b>(0.3918,1)</b>



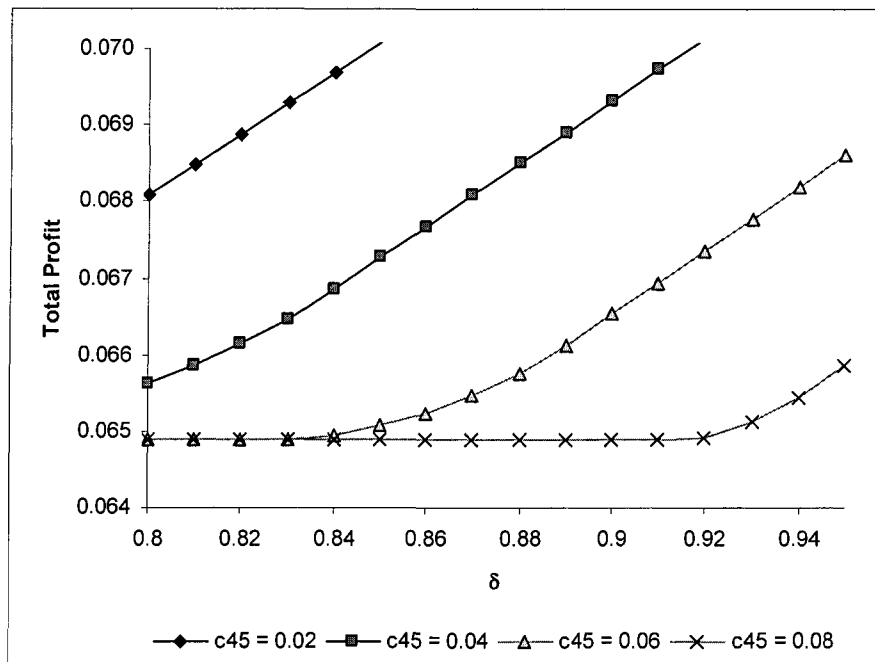
### **4.1.1 Cost and Quality of Refurbished Products**

Although the quality of refurbished products might be improved by investing in more costly refurbishing processes, this relationship is difficult to measure as the quality also depends on company policy (see section 1.1) and consumer perception. Figure 7 shows the effects of  $\delta$  and  $c_{45}$  on the optimal objective function value and decision variables, when the two quantities are varied independently. Increasing  $\delta$  increases the total profit,  $p_{mr}^*$ , and  $P_{ref}^*$  because refurbished products will enjoy higher demand and merit higher prices. The opposite effects result from increasing  $c_{45}$  because the higher cost reduces the profit from selling refurbished products. Note that a shift from  $p_{mr} = 0$  to a positive value is accompanied by a discontinuous drop in  $P_{ref}$ . In cases where  $p_{mr}^* = 1$ ,  $P_{ref}^*$  does not necessarily increase with  $c_{45}$  because, while the profit margin for refurbished products decreases, the profit from new products is unchanged. It may be more profitable overall to sacrifice profit from refurbished products in favor of higher demand for new products instead of increasing  $P_{ref}$  to recover the cost.

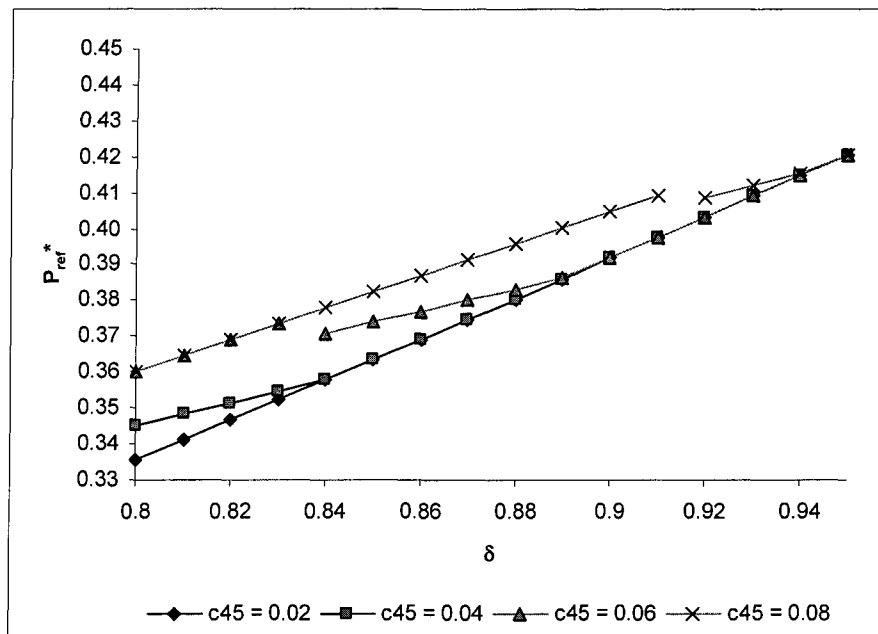
### **4.1.2 Price and Backorder Cost for New Products**

The price of new products, determined in the competitive market, affects both the market share as measured by the demand rate and the marginal profit from new products. A low value of  $P_{new}$  creates a scenario where the manufacturer sells a high volume of products with a low profit margin. If the price is very low, the new product demand may exceed the manufacturing capacity. We varied  $P_{new}$  from 0.30, enough higher than the manufacturing

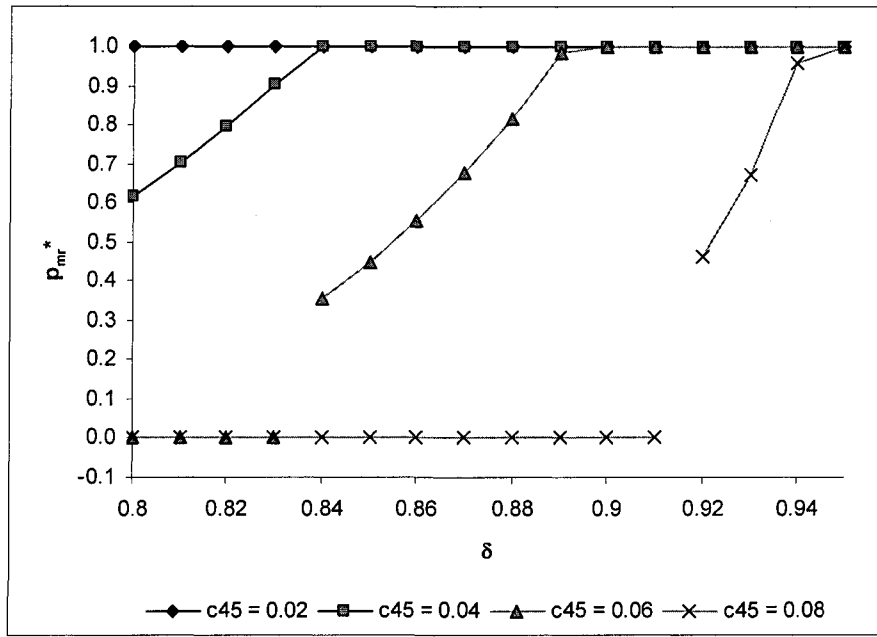
cost to generate profit, to 0.95, less than one to guarantee some demand. Figure 8a shows the feasible region when  $P_{new}$  is between 0.3 and  $0.4 = 1 - \mu_1$ . In this case, constraint (3) forms part of the feasible region's boundary because any value of  $P_{ref} \geq P_{new} - (1 - \delta)(1 - \mu_1)$  would cause demand for new products to exceed the manufacturing capacity, resulting in infinite backorders at the manufacturing station. Instead, the price of refurbished products must be set low enough to decrease the demand for new products and create demand for refurbished products instead. Feasible values of  $p_{mr}$  do not include 0. When  $p_{mr}$  is large,  $P_{ref}$  is also bounded above by constraint (6) to prevent an exploding inventory of refurbished products. When  $P_{new} > 0.4$ , the manufacturing site has enough capacity to process all possible demands for new products. In this case,  $p_{mr}$  may take on any value between 0 and 1 and only the stability of the queue of refurbished inventory at station 5 places an upper bound of  $P_{ref}^*$  as shown in Figure 8b.



(a)

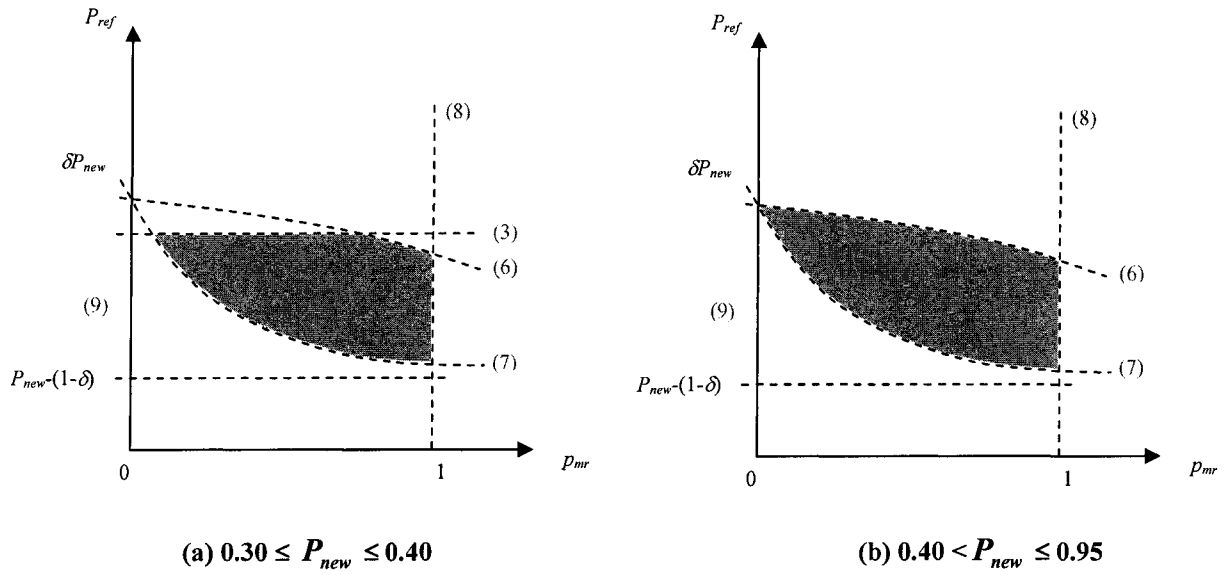


(b)



(c)

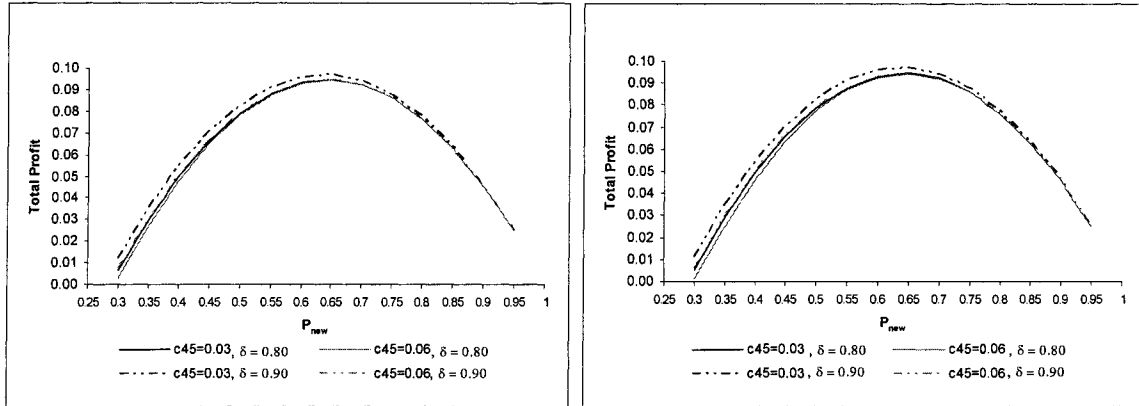
FIGURE 7. Total profit,  $P_{ref}^*$  and  $p_{mr}^*$  for different values of  $\delta$  and  $c_{45}$  at  $P_{new} = 0.45$  and  $h_1 = 0.0001$



(a)  $0.30 \leq P_{new} \leq 0.40$

(b)  $0.40 < P_{new} \leq 0.95$

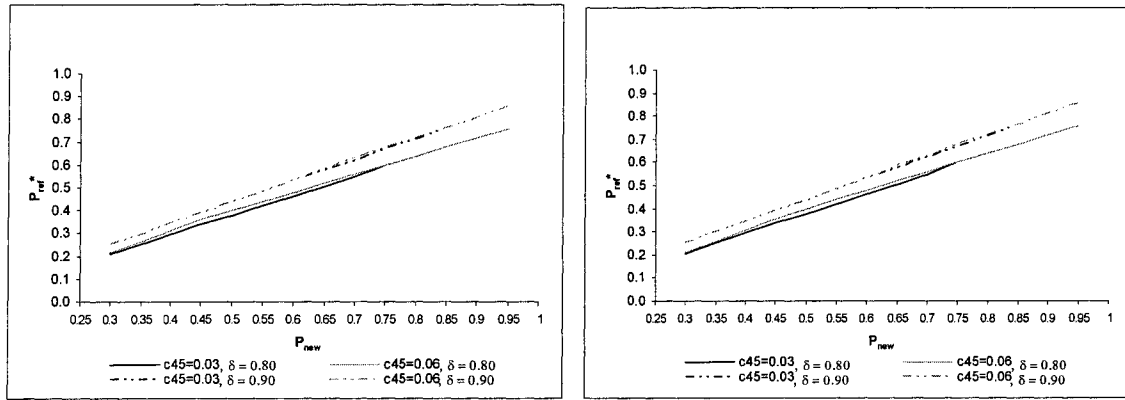
FIGURE 8. Feasible regions with two different sets of constraints depending on  $P_{new}$ .



(a)  $h_1 = 0.00005$

(b)  $h_1 = 0.00020$

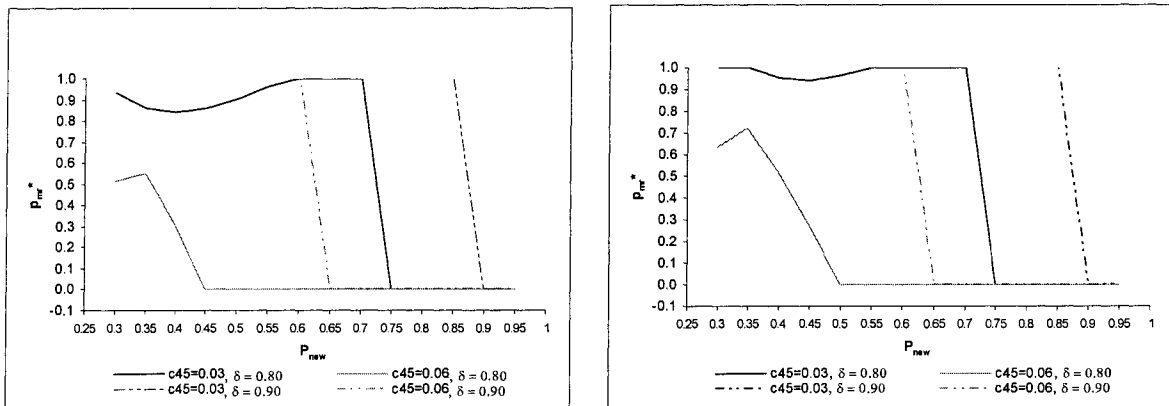
FIGURE 9. The optimal total profit for different  $P_{new}$ ,  $h_1$ ,  $c_{45}$  and  $\delta$ .



(a)  $h_1 = 0.00005$

(b)  $h_1 = 0.00020$

FIGURE 10. The optimal price for refurbished products for different  $P_{new}$ ,  $h_1$ ,  $c_{45}$  and  $\delta$ .



(a)  $h_1 = 0.00005$

(b)  $h_1 = 0.00020$

FIGURE 11. The optimal proportion of returns to refurbish for different  $P_{new}$ ,  $h_1$ ,  $c_{45}$  and  $\delta$ .

The cost of new product backorders is another important parameter that can be difficult to quantify. We expect that higher values would also encourage refurbishing as a way to increase customer satisfaction. Figures 9, 10 and 11 show total profit,  $P_{ref}^*$  and  $p_{mr}^*$  at different values of  $P_{new}$ ,  $h_1$ ,  $c_{45}$  and  $\delta$ . The total profit is concave with respect to  $P_{new}$  (see proof in Appendix 3) such that the highest total profit is achieved under similar values of  $P_{new}$  for a variety of combinations of the other parameters. Figure 10 shows that  $P_{ref}^*$  predictably increases with  $P_{new}$ . When  $P_{new}$  is large, its increase reduces the optimal  $p_{mr}$  because of the higher profit margin for new products; however, the optimal proportion to refurbish exhibits varied behavior when the price of new products is low, particularly when it is low enough that new product demand exceeds capacity. Moreover at low  $P_{new}$ ,  $p_{mr}^*$  increases when backorder cost is higher (Figure 11) because refurbished products ease the demand for new products and subsequently lower the number of orders waiting in the manufacturing site. This effect is less significant as  $P_{new}$  increases because the number of orders waiting in the manufacturing site decreases. The effect of backorder cost on total profit and  $P_{ref}^*$  is not obviously seen because of the vast difference in total profit and  $P_{ref}^*$  for different  $P_{new}$ . Table 5 summarizes the effects of parameters  $P_{new}$ ,  $h_1$ ,  $\delta$  and  $c_{45}$  on the total profit,  $p_{mr}^*$ , and  $P_{ref}^*$ . Generally, refurbishing is encouraged by high perceived quality achieved at a low refurbishing cost, and/or high backorder costs with a low price for new products. The optimal refurbished product price increases with higher quality and lower back order cost. The cost of refurbishing has both negative and positive effects on price of refurbished products. When only some of returned products are refurbished, increasing the

cost of refurbishing causes higher optimal price and consequently lower quantity of refurbished products. In contrast, when all returned products are refurbished, the optimal price of refurbished products decreases as its cost increases. Since the quantity of refurbished products remains unchanged, increasing price will incur high inventory cost. We believe this surprising result occurs because the inventory cost at the refurbishing station outweighs the benefit from higher marginal profit.

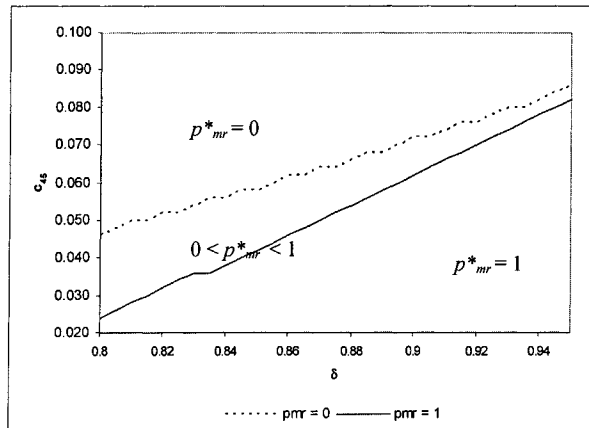
TABLE 5. The effect of parameters  $P_{new}$ ,  $h_1$ ,  $\delta$  and  $c_{45}$  on total profit,  $p_{mr}^*$ , and  $P_{ref}^*$

	Total Profit	$p_{mr}^*$	$P_{ref}^*$		Sensitivity of the optimal policy
			$0 < p_{mr} < 1$	$p_{mr} = 1$	
$\delta$	pos.	pos.	pos.	pos.	pos.
$c_{45}$	neg.	neg.	pos.	neg.	pos.
$P_{new}$	concave	varies.	pos.	pos.	pos.
$h_1$	neg.	pos.	neg.	neg.	neg.

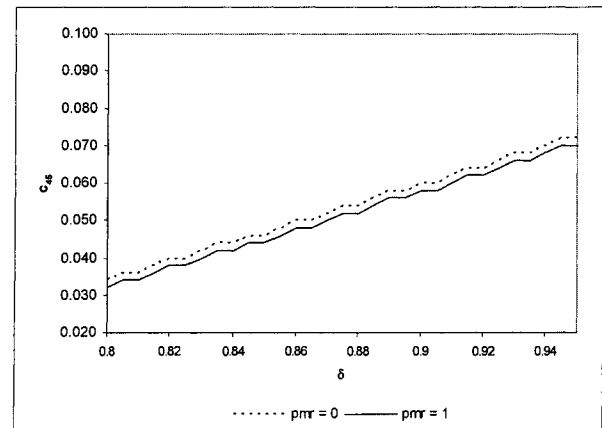
#### 4.1.3 Sensitivity of the Optimal Quantity of Refurbished Products

Given that it is optimal to refurbish either none or a significant proportion of the returned products, it is important to understand conditions under which small changes in parameters cause a shift from one paradigm to the other. Figure 12 shows the sensitivity of the optimal proportion to refurbish. The cost of refurbishing products  $c_{45}$  ranges from 0.02, or 8% of the manufacturing cost, to 0.10, beyond which  $p_{mr}^* = 0$  in most cases regardless of the quality. The quality parameter  $\delta$  ranges from 0.80 to 0.95 to focus on commercial product returns, which still preserve most but not all of the perceived quality of new

products. In Figure 12a, when refurbishment cost and quality of refurbished products increases,  $p_{mr}^*$  shifts abruptly from zero to one; in other words, the optimal policy is more sensitive to the small changes in parameter values. This effect is less significant when the backorder cost is higher as in Figure 12c. In Figures 12b and 12d, when  $P_{new}$  is 0.65 or near where the maximum profit can be achieved (from the previous section),  $p_{mr}^*$  is very sensitive to changes in refurbishment cost and quality of refurbished products and the backorder cost becomes almost irrelevant except at the lowest value for  $\delta$  shown in Figure 12d. Table 5 summarizes these results. It is worthwhile to carefully assess the market for refurbished products before deciding whether to offer them. But when high demand for new products and stiff backorder penalties combine with low refurbishment cost and less perceived quality, refurbishment is less of an all-or-nothing proposition.



(a)  $P_{new} = 0.45$  and  $h_1 = 0.00005$



(b)  $P_{new} = 0.65$  and  $h_1 = 0.00005$



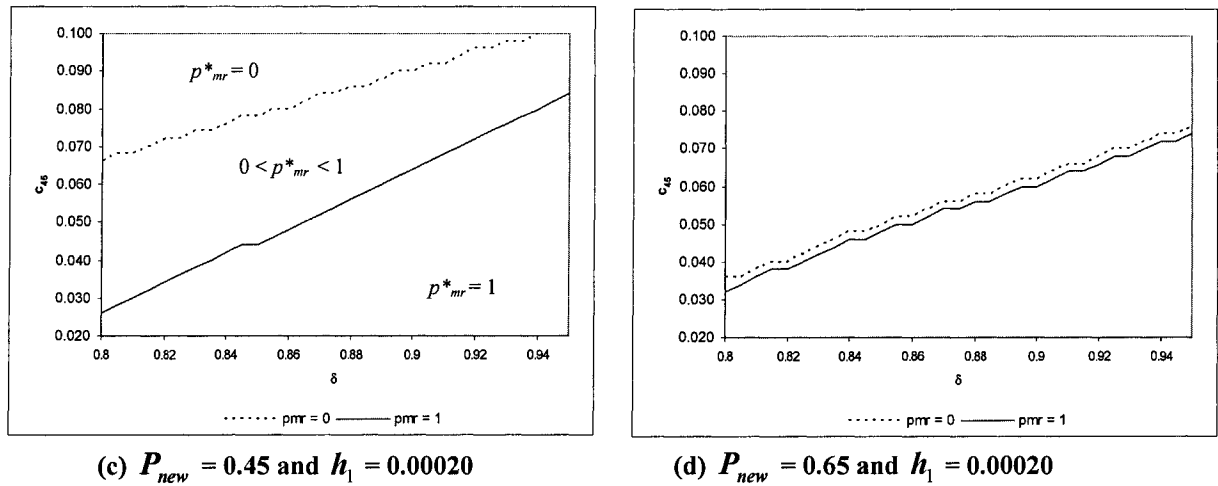


FIGURE 12. The response of  $p_{mr}^*$  to different  $P_{new}$ ,  $h_1$ ,  $\delta$  and  $c_{45}$

## 4.2 General Queueing Network Model

The general queueing network expands the capability of the model by allowing us to set the variability of service times, keeping track of grades and return histories and varying the policy of the refurbishing site.

### 4.2.1 Effect of Variability

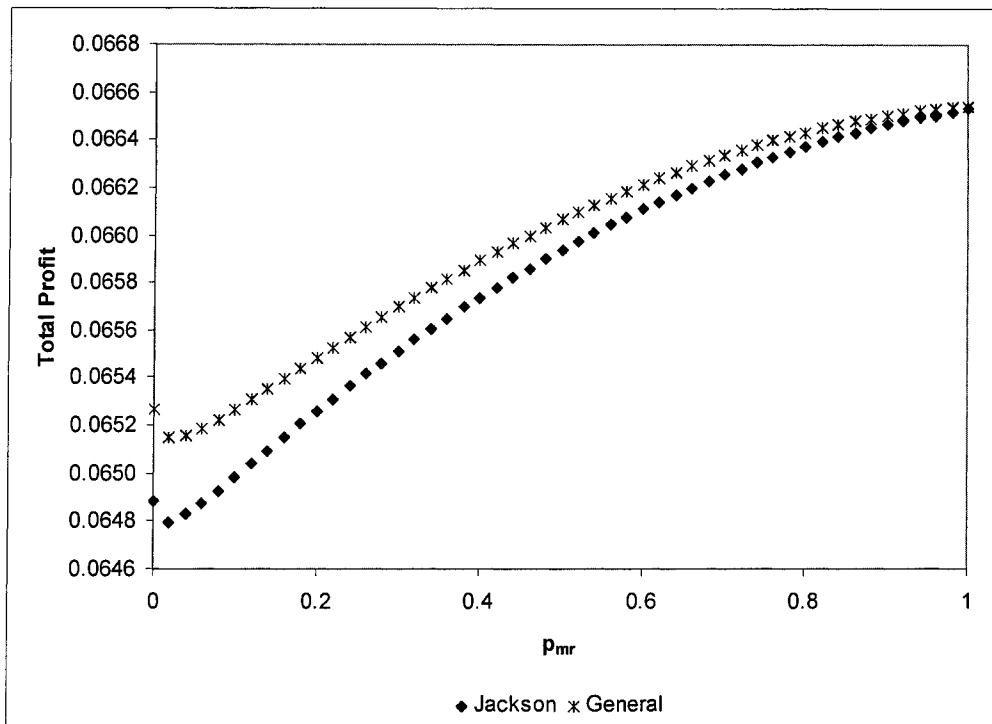
In the Jackson queueing network model, with exponentially distributed service times, the variance of service time equals the square of the mean. In other words, the scv is equal to one. This variability is not a good representation of the real process. The previous study by Ketzenberg and Souza (2003) shows that the scv of a new product assembly line is in the range of 0 to 1 while the scv of a remanufacturing line ranges from 0.5 to 2.0. Souza and Ketzenberg (2002) state that the variability of the service time at the evaluation center is very minimal. In addition, the time products are in consumers' hands is limited by the return period, therefore the exponential service time is not a good representation and  $C_{s_2}^2$  should be

less than one. Nevertheless, the exponential distribution can well represent the interarrival time of demands at station 1 that are independent of each other. The service time at station 5 also remains exponentially distributed to preserve the memoryless property so that the time between demand arrivals is independent of the availability of refurbished products. To focus solely on the effect of variability, we assume job grades and types are aggregated so that the flow rates are exactly the same as in the Jackson network model, the service protocols are FCFS for all stations, and we do not limit the maximum number of times products can be refurbished. Therefore, only the variability of service time distinguishes this model from the Jackson queueing network model in Section 4.1. Details of this model can be found in case 3 of Appendix 1. Note that this general open queueing network with no priority yields the identical results to the Jackson open queueing network model when  $C_{s_i}^2 = 1, i = 1, \dots, 5$ .

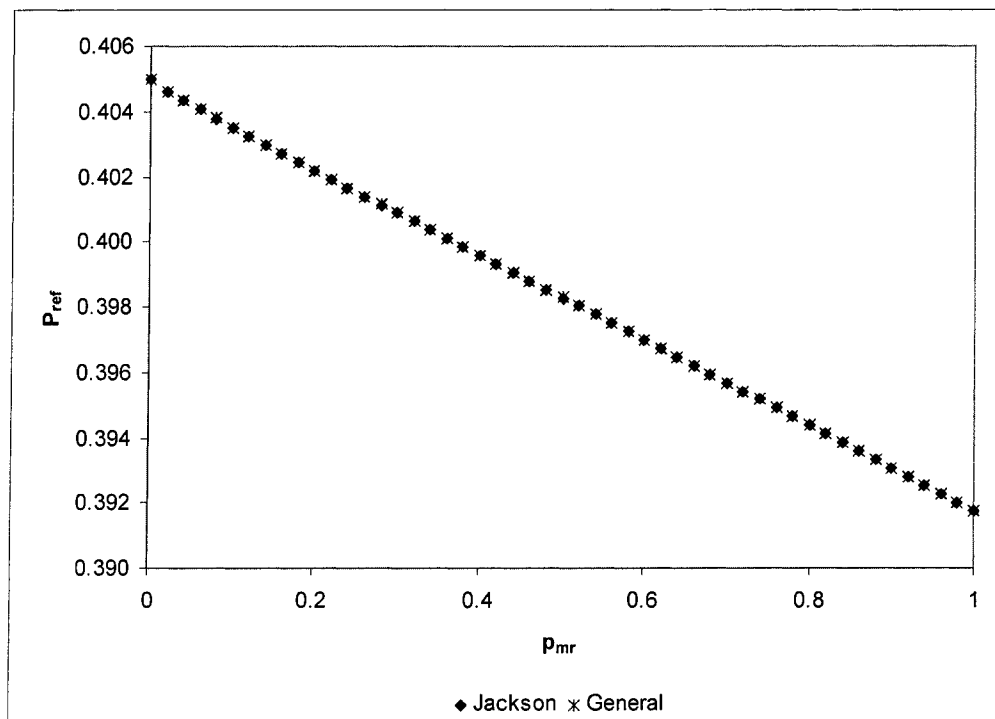
Parameters  $P_{new}$ ,  $c_{45}$  and  $h_1$  are held constant in order to focus on the effects of variability and the refurbishing policy. Following are the constant parameter values in this section:  $P_{new} = 0.45$ ,  $P_{dis} = 0.15$ ,  $p_{cr} = 0.25$ ,  $c_{01} = c_{23} = c_{20} = c_{40} = c_{52} = 0$ ,  $c_{12} = 0.25$ ,  $c_{30} = 0.02$ ,  $c_{34} = 0.01$ ,  $c_{45} = 0.06$ ,  $h_1 = 0.0001$ ,  $h_2 = 0$ ,  $h_3 = 0.00005$ ,  $h_4 = 0.00005$ ,  $h_5 = 0.00005$ ,  $\mu_1 = 0.6$ ,  $\mu_2 = 0.006$ ,  $\mu_3 = 0.3$ ,  $\mu_4 = 0.6$ . For the general case, the scv of service time at each station is assigned as follows:  $C_{s_1}^2 = 0.25$ , which is in the range of scv of assembly line in Knott (1987).  $C_{s_2}^2 = \left( \frac{2E[S_2]^2}{12} \right) / E[S_2]^2 = 0.33$  by assuming that the time a product will be held by the consumer has a uniform distribution with parameters  $(0, 2E[S_2])$ .  $C_{s_3}^2 = 0$  and  $C_{s_4}^2 = 1.7825$  according to the study by Souza and Ketzenberg (2002). They state that at the evaluation center the variability is very minimal while at

remanufacturing station, the variability is in the range of 1.5 to 2.  $C_{s_5}^2 = 1$  because the service time at station 5 represents the exponentially distributed interarrival time of demands for refurbished products.

Figure 13 compares the total profit and optimal price of refurbished products from the Jackson and general models. While the optimal price of refurbished products from the two cases are almost the same ( $\pm 0.006\%$ ), the difference in total profit is more evident at low values of  $p_{mr}$ . Since there is no significant difference in the optimal prices of refurbished products, we might ignore the difference in revenue and focus our analysis on the inventory cost. The variability of service times directly affects the number of jobs in queue: The low variability of service time reduces the number of jobs in queue and high variability increases it. In the general model, the newly assigned variability results in lower backorder and inventory costs at stations 1, 2 and 3 but higher inventory cost at station 4. Since there is no holding cost at station 2, and station 3 has plenty of capacity to evaluate returned products, the primary inventory cost occurs at stations 1 and 4. When the manufacturer refurbishes only a small portion of returned products (low  $p_{mr}$ ), the demand for new products is high. At this point, the manufacturer benefits from the low variability of the service time at station 1. On the other hand, as the quantity of refurbished products increases, the variability at station 4 becomes more important. The profit comparison in Figure 13 demonstrates that the cost savings from low inventory at station 1 outweigh the higher inventory cost at station 4 for the entire range of  $p_{mr}$ .

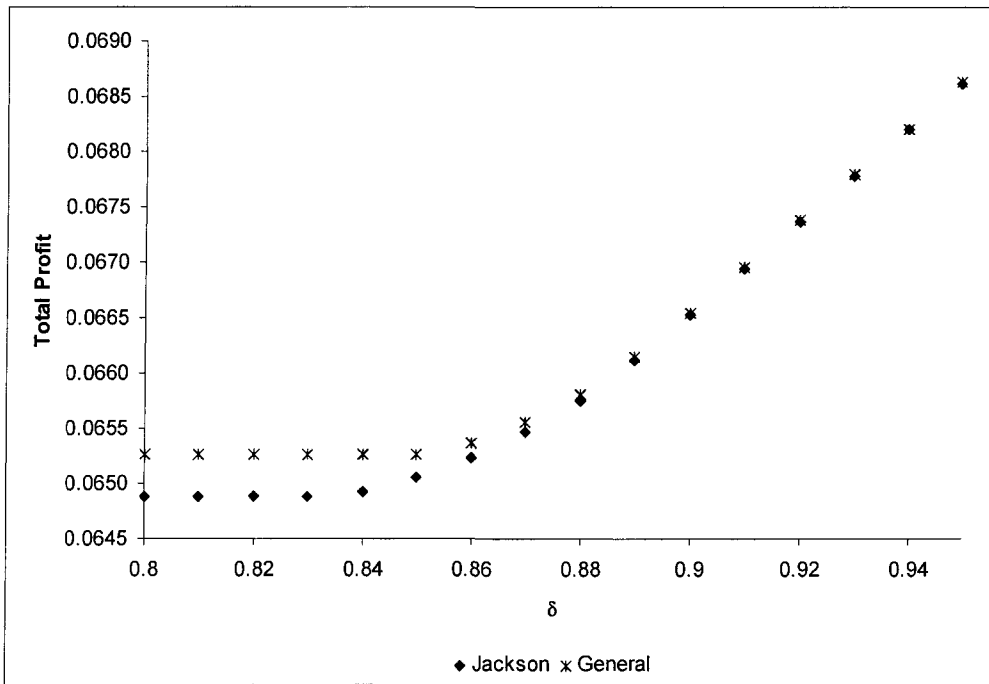


(a)

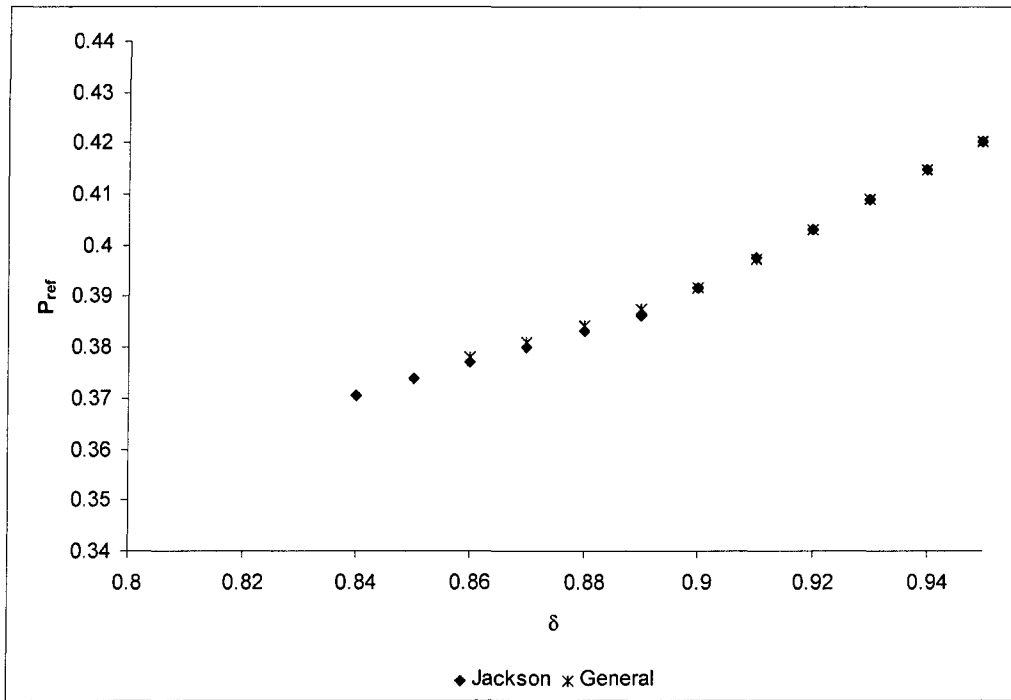


(b)

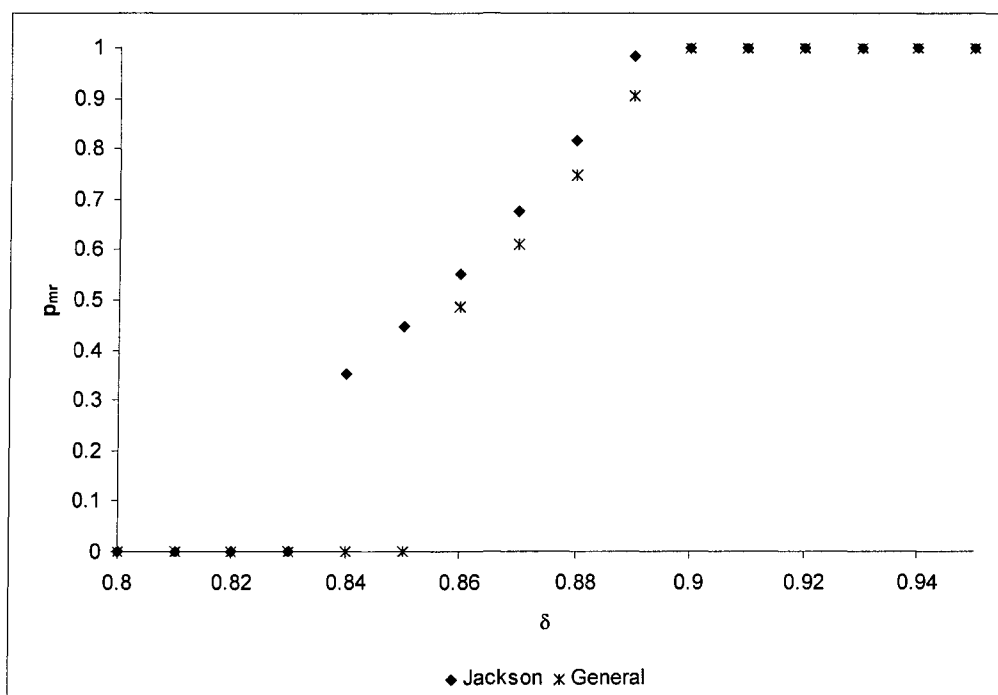
FIGURE 13. Total profit and price of refurbished products from Jackson open queueing and general open queueing models at  $\delta = 0.9$ .



(a)



(b)



(c)

Figure 14. Total profit,  $P_{ref}^*$  and  $P_{mr}^*$  from the Jackson open queueing and the general open queueing networks for different quality levels.

Figure 14 shows how the optimal solution and total profit change as the quality of refurbished products increases. The minimum quality level where refurbishing some of returned products yields higher profit than dismantling all of them shifts from  $\delta = 0.84$  in case of the Jackson queueing network to  $\delta = 0.86$  in case of the general queueing network. High variability at station 4 in the general model results in a lower proportion of returned products to be refurbished.

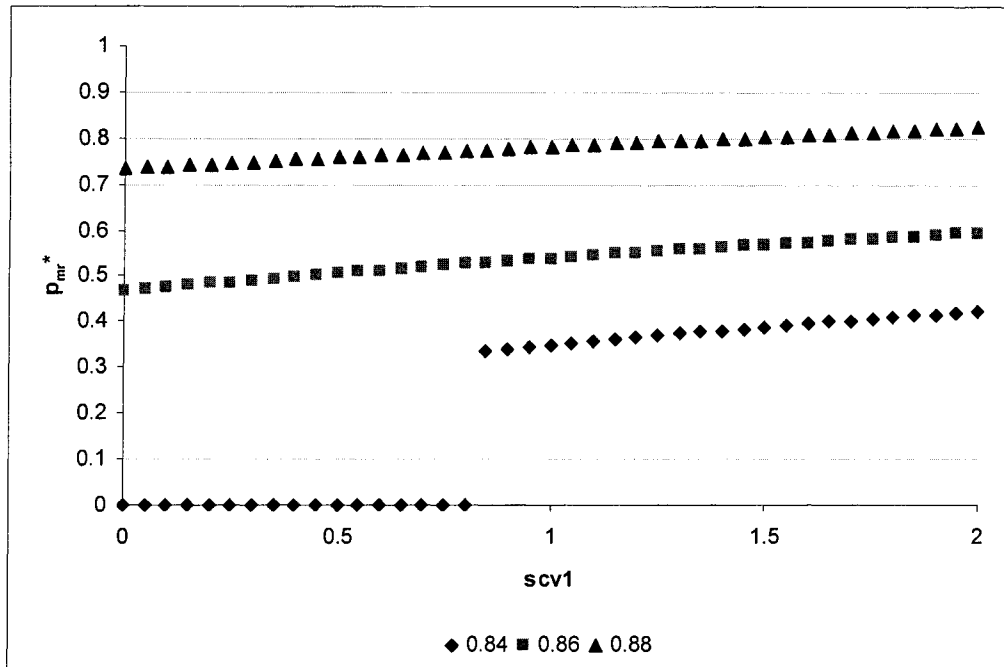
In a queueing system where jobs arrive and are served one at a time, the number of jobs in queue is induced by the variability of service time. Therefore, higher service time variability at stations where the manufacturer is responsible for the inventory costs results in less total profit. To study the effect of each station on the decision variables, the scv of service times of a particular station is varied from 0 to 2 while scv of other stations are held

constant  $\{C_{s_1}^2 = 0.25, C_{s_1}^2 = 0.33, C_{s_1}^2 = 0.00, C_{s_1}^2 = 1.7825, C_{s_5}^2 = 1\}$ . Table 6 summarizes the effects of variability of stations 1 to 4 on total profit and the decision variables.

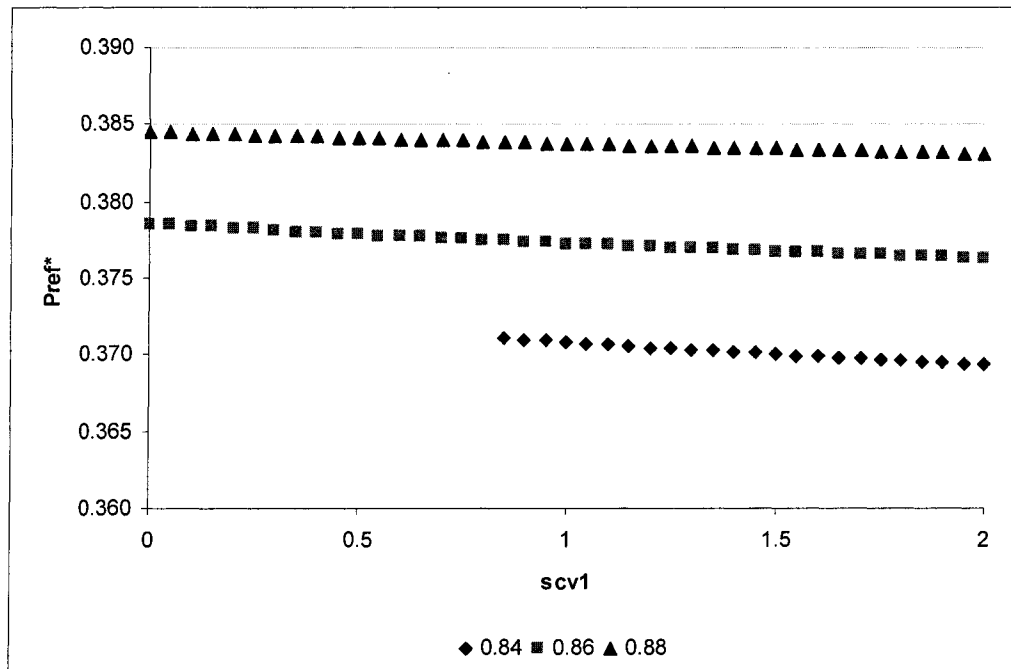
**TABLE 6.** The effect of variability at station 1 to 4 on total profit,  $p_{mr}^*$ , and  $P_{ref}^*$ .

Variability at	Total Profit	$p_{mr}^*$	$P_{ref}^*$	
			$0 < p_{mr} < 1$	$p_{mr} = 1$
<b>Station 1</b>	<b>neg.</b>	<b>pos.</b>	<b>neg.</b>	<b>neg.</b>
Station 2	no effect	no effect	no effect	no effect
Station 3	neg.	no effect	pos.	neg.
<b>Station 4</b>	<b>neg.</b>	<b>neg.</b>	<b>pos.</b>	<b>neg.</b>

We observe that variability at station 1 and 4 have significant effects on total profit and the decision variables as shown in Figures 15 and 16. At station 1, higher variability at the manufacturing station causes higher  $p_{mr}^*$  and lower  $P_{ref}^*$  as shown in Figure 15a and 15b, respectively. In contrast, higher variability at the refurbishing station causes lower  $p_{mr}^*$  and higher  $P_{ref}^*$ . The reason is that variability of products causes jobs to wait in the queue. When variability of service time in either manufacturing or refurbishing is high, it is optimal to reduce orders of new products at the manufacturing station to avoid backorder cost. Similarly, at the refurbishing station it is optimal to reduce the quantity of refurbished products to avoid high inventory costs. Figures 15 and 16 illustrate how the decision variables change to adjust the demands.



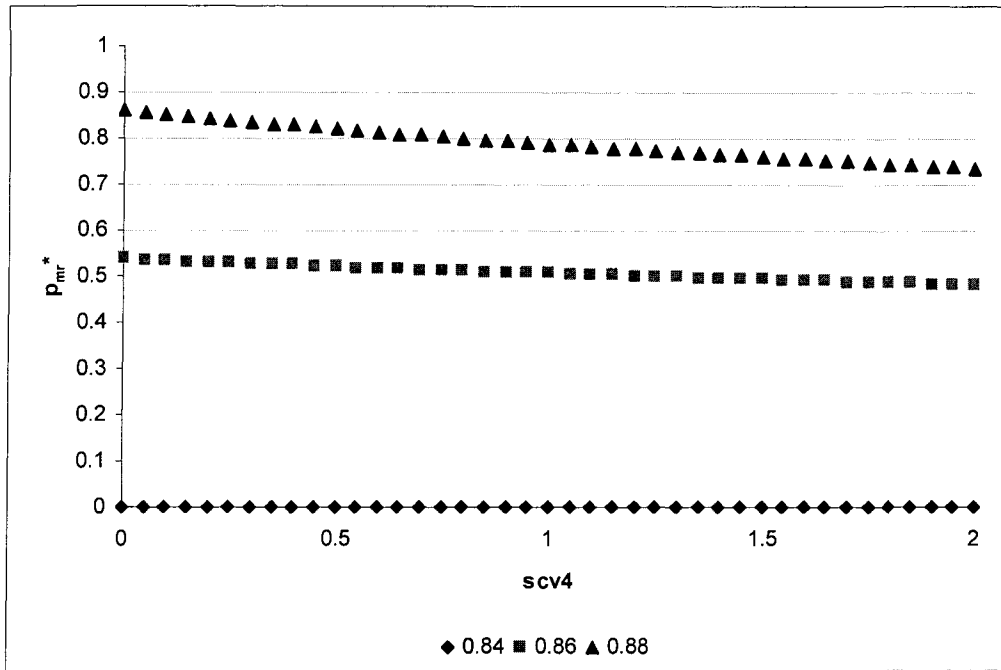
(a)



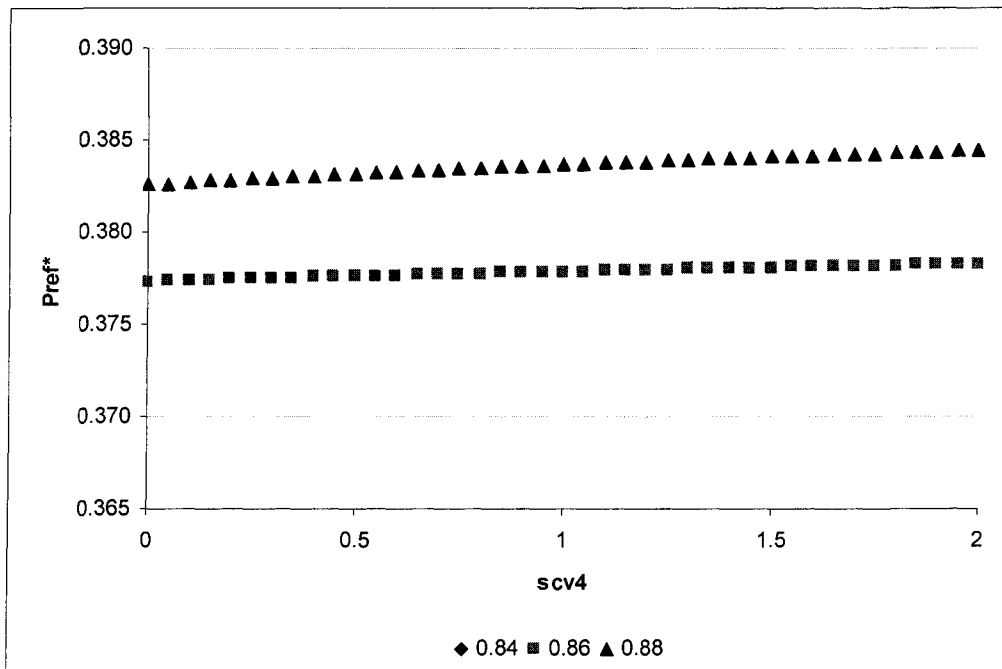
(b)

Figure 15. Effect of variability of station 1 service times on  $p_{mr}^*$ ,  $P_{ref}^*$  and total profit for different levels of perceived refurbished item quality





(a)



(b)

Figure 16. Effect of variability of station 4 service times on  $p_{mr}^*$ ,  $P_{ref}^*$  and total profit for different levels of perceived refurbished item quality.

The variability at station 2 has no effect on total profit and  $(P_{ref}^*, p_{mr}^*)$  because of 2 reasons: 1) the inventory cost at station 2 is not a responsibility of the manufacturer,  $h_2 E[N_2] = 0$  and 2) the variability at station 2 does not affect the variability of arrival times of other stations since the effect of variability at station 2 is nullified as  $\rho_2 \rightarrow 0$  and  $m \rightarrow \infty$ ,

$$c_{d_2}^2 = \lim_{\rho_2 \rightarrow 0, m_2 \rightarrow \infty} \left( 1 + (1 - \rho_2^2)(c_{a_2}^2 - 1) + \frac{\rho_2^2(c_{s_2}^2 - 1)}{\sqrt{m_2}} \right) = c_{a_2}^2. \text{ Similarly, the effect of variability at}$$

station 3 on total profit and  $(P_{ref}^*, p_{mr}^*)$  is very small because of the small value of  $\rho_3$  that leads to low inventories and low effect on variability of arrival times of other stations.

#### 4.2.2 Evaluating and Sorting Returned Products

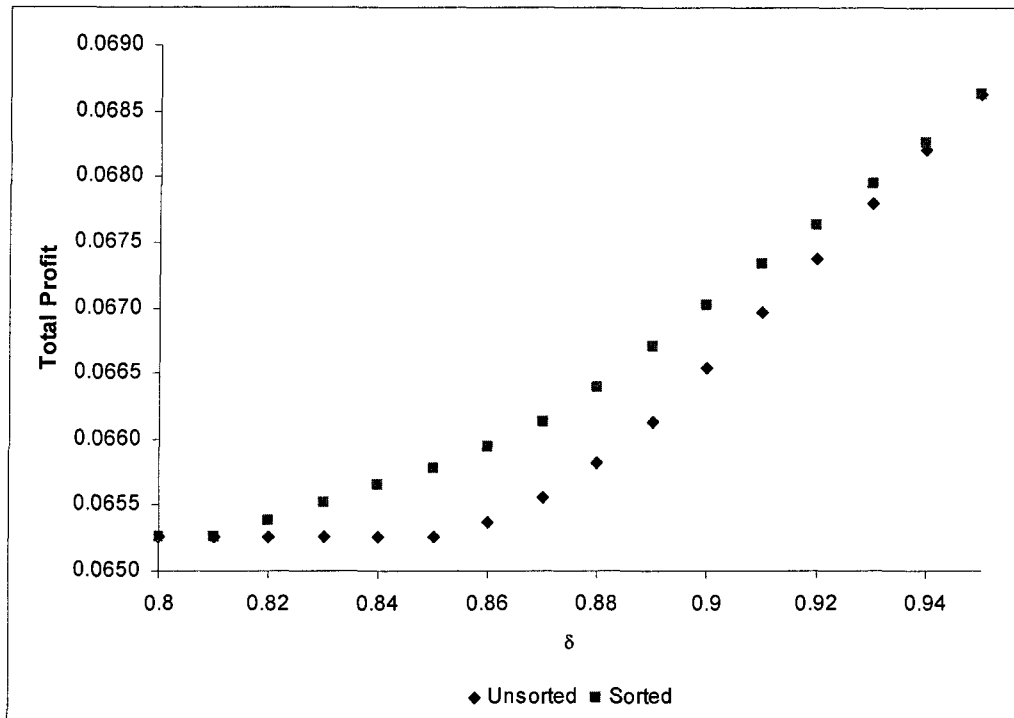
Returned products from consumers are inspected at the evaluation center and then sent to be either refurbished or dismantled. In this stage, instead of considering only the quantity, the manufacturer can consider grades of returned products and decide on the optimal proportion to refurbish based on their grades. In this section, the refurbishing policy is still FCFS and the number of times products can be refurbished is unlimited.

Let  $p_{mr}^{(\bullet r)}$  be the probability that returned products of grade  $r$  will be sent to be refurbished,  $r = 1, \dots, \theta$ . If  $\theta = 3$ , accordingly, we have decision variables  $p_{mr}^{(\bullet 1)}$ ,  $p_{mr}^{(\bullet 2)}$  and  $p_{mr}^{(\bullet 3)}$ , where  $\sum_{r=1}^3 p_{mr}^{(\bullet r)} = p_{34}$  and  $p_{mr}^{(\bullet r)} = p_{mr}^{(l,r)}$  for  $r = 1, \dots, 3$ ,  $l = 1, \dots, \infty$ . Product grade 1 requires less time and cost to refurbish than grade 2. Likewise, grade 2 requires less time and cost than grade 3. Therefore, the priority is given to products with less time and cost

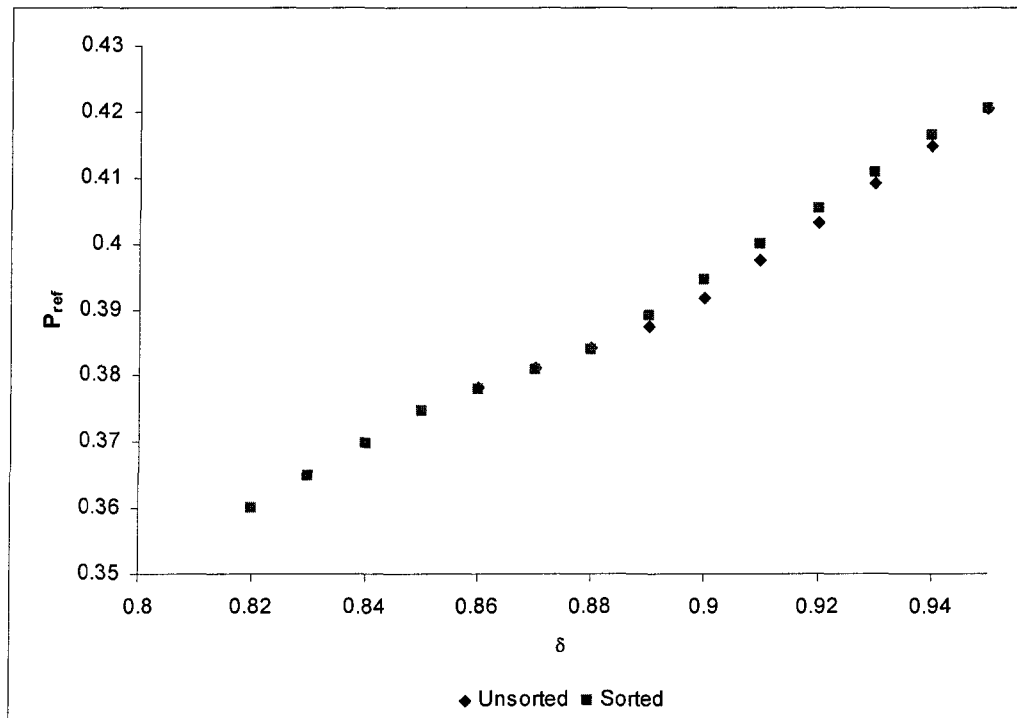
required to refurbish. As discussed in Section 3.3, we reduce the complexity of our model by considering the variables  $p_{mr}^{(\bullet r)}$  one at a time. Other grades with higher and lower priority than grade  $r$  have  $p_{mr}^{(\bullet r)}$  values of one and zero, respectively. The model is a special case of the general open queueing model where types of jobs are aggregated and priority is assigned only to grades. More details of the optimization model and its convexity can be found in case 2 of Appendix 1.

We set  $\theta = 3$ ,  $q^{(\bullet 1)} = 0.4$ ,  $q^{(\bullet 2)} = 0.4$ ,  $q^{(\bullet 3)} = 0.2$ , and  $P_{dis}^{(\bullet 1)} = 0.16$ ,  $P_{dis}^{(\bullet 2)} = 0.15$ ,  $P_{dis}^{(\bullet 3)} = 0.13$ ,  $c_{34}^{(\bullet 1)} = 0.04$ ,  $c_{34}^{(\bullet 2)} = 0.06$ ,  $c_{34}^{(\bullet 3)} = 0.10$ ,  $E[S_4^{(\bullet 1)}] = 2.5$ ,  $E[S_4^{(\bullet 2)}] = 3.5$ ,  $E[S_4^{(\bullet 3)}] = 4.67$ . These parameters are set so that the weighted average of parameters  $P_{ref}^{(\bullet r)}$ ,  $c_{34}^{(\bullet r)}$ ,  $E[S_4^{(\bullet r)}]$ ,  $c_{s_4}^{2(\bullet r)}$  are equal to parameters from the case in Section 4.2.1. The proportions of grades were obtained from the working paper of Ferguson et al. (2006), which shows the percentage of products returned based on reason. 20% of returned products are true failure products, which are categorized in grade 3. We further categorize the rest of returned products into grade 1 and 2 in equal proportions by assuming that the rest of the returned products do not have uniform quality because of different amounts of wear or flaws that might occur from consumer usage. A good example of the difference between product grade 1 and 2 is the number of pages printed from returned printer (Souza et al., 2006). If the number of pages is below some threshold, the printer can be repackaged and resold as a new product. We assume that grade 2 requires additional refurbishing processes so that it is different from grade 1. Therefore, products with better grade require less cost, time and variability of refurbishing time.

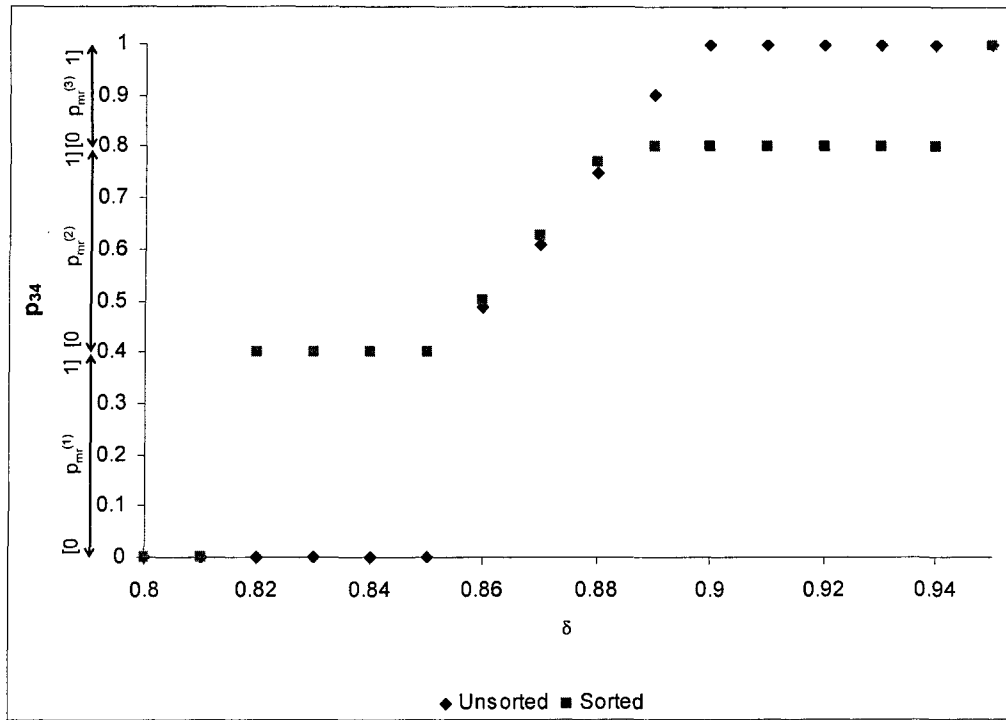
The results show that sorting products is an option to improve total profit (see Figure 17). Figure 17c shows differences in the optimal quantity of refurbished products. When sorted by grade, it is more profitable to send all returned products with grade 1 to be refurbished at perceived quality of 0.82 while in the unsorted case it is not profitable until the quality reaches 0.86. In contrast, grade 3 products will be refurbished only when quality goes up to 0.95 as opposed to sending all returned products to refurbish when quality is 0.90 in the unsorted case. The optimal price of refurbished products changes with the quantity; at the same quality, the price is lower when quantity is higher as in Figures 17b and 17c. Figure 15a shows the higher total profit when products are sorted. However, the evaluation does not affect the total profit when either none or all returns are sent to be refurbished. At quality levels of 0.89 to 0.94 in Figures 17a and 17c, it is interesting to see that the higher quantity of refurbished products does not necessary result in higher total profit. The optimal proportion of returned products to refurbish also depends on their grades. When some of the returned products are refurbished, sorted returned products consist of better grade products compared with unsorted at the same quantity. The better proportion helps the manufacturer refurbish products with less cost and variability in the refurbishing time. The result is higher total profit as shown in Figure 17. For example, if 40% of returned products are refurbished then, when sorted, only grade 1 products will be refurbished, as opposed to a mixture of all grades (40% of grade 1, 2 and 3) when they are unsorted.



(a)



(b)



(c)

Figure 17.  $P_{mr}^*$ ,  $P_{ref}^*$  and total profit from the general open queueing model with sorted and unsorted grades of returned products for different quality levels of refurbished products.

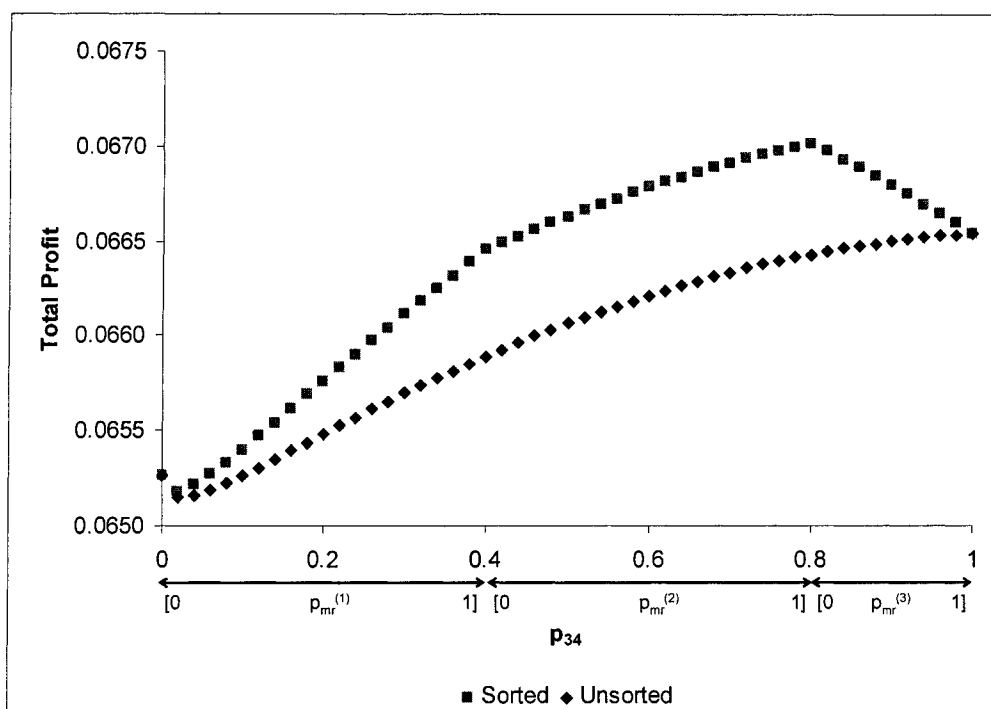
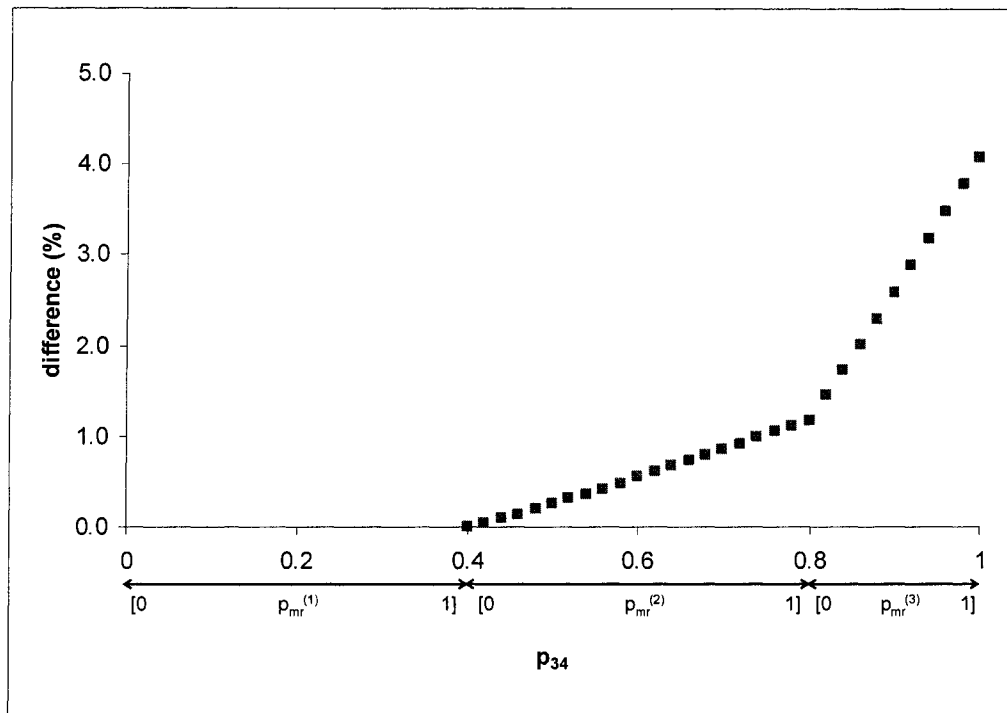


Figure 18. Total profit at  $P_{ref}^*$  with different quantities of refurbished products from Unsorted and Sorted cases at  $\delta = 0.9$ .

### 4.2.3 SEPT Policy

The model in section 4.2.2 assumes the FCFS discipline in the refurbishing station despite the different grades of products. As we discussed in Chapter 3, the SEPT service protocol is generally known to reduce the number of jobs in queue compared to FCFS in many situations. We compare the performance of these two service protocols in the general open queueing model with grade priority with the same parameters as in previous sections. The result shows no difference in optimal values of the decision variables  $P_{ref}$  and  $p_{mr}^{(r)}$ ,  $r = 1, \dots, 3$ . However, SEPT achieves a slightly higher total profit than FCFS because of the lower inventory cost due to fewer jobs in queue at station 4. Because the inventory cost saving is very minimal compared to total profit, Figure 19 shows the results in percent difference of inventory cost at the refurbishing station.



**Figure 19. Percent difference in inventory cost at station 4 achieved by SEPT rather than FCFS policy**

There is no difference in inventory cost between SEPT and FCFS when refurbishing only grade 1 products since no priority is involved. The difference begins at the point where grade 2 products are also refurbished and increases with the quantity of product grades 2 and 3 refurbished. The benefit of SEPT is more obvious when the difference between products is greater. Because SEPT yields the higher total profit, from this point we use only SEPT to perform the rest of experiments.

#### ***4.2.4 The Number of Times Products Can Be Refurbished***

In this section, the model tracks not only grades but also the number of times products have been returned. Let  $\eta$  = number of times products can be refurbished. We compare the optimal prices and quantities of refurbished products when  $\eta = 1, 2$  and infinity. The model is a general network with grade and number of returns priority. Details of the optimization



model can be found in case 1 of Appendix 1. Note that since  $\eta$  in this section is finite,  $p_{34}$

$$\text{is now a function of } \eta: p_{34} = \frac{\lambda_{34}}{\lambda_3} = \frac{\lambda_{new} \left( \sum_{l=1}^{\eta} \prod_{k=1}^l p_{cr} \sum_{r=1}^{\theta} (p_{mr}^{(k,r)} q^{(k,r)}) \right)}{\lambda_{new} p_{cr} \left( 1 + \sum_{l=1}^{\eta} \prod_{k=1}^l p_{cr} \sum_{r=1}^{\theta} (p_{mr}^{(k,r)} q^{(k,r)}) \right)}. \text{ Here } p_{34} \text{ has}$$

the value less than one since  $\lambda_3 > \lambda_{34}$  when  $\eta < \infty$ . Therefore it is impossible to send all returned products to be refurbished since some of them can no longer be refurbished. Recall that  $p_{34}$  is a proportion of returned products sent to be refurbished. The quantity of refurbished products is now represented by  $\lambda_4$ . A higher value of  $\eta$  will result in a higher quantity of refurbished products at the same level of  $p_{34}$ .

We first study the difference between the two priority policies: grade priority and grade-type priority. With grade priority, products with the same grade have the same priority independent of their types; in other words, independent of the number of times returned. For example, if  $\eta = 2$ ,  $\theta = 3$ , then class  $(1,1) = (2,1) \succ (1,2) = (2,2) \succ (1,3) = (2,3)$ . With grade-type priority, products with fewer returns will have the higher priority. For example, if  $\eta = 2$ ,  $\theta = 3$ , then class  $(1,1) \succ (2,1) \succ (1,2) \succ (2,2) \succ (1,3) \succ (2,3)$ . With assumption that grades are independent of the number of times returned, the parameters are the same as in the previous section except for the following: for  $l = 1, \dots, \eta$ ,

$$q^{(l,1)} = 0.4, q^{(l,2)} = 0.4, q^{(l,3)} = 0.2,$$

$$P_{dis}^{(l,1)} = 0.16, P_{dis}^{(l,2)} = 0.15, P_{dis}^{(l,3)} = 0.13,$$

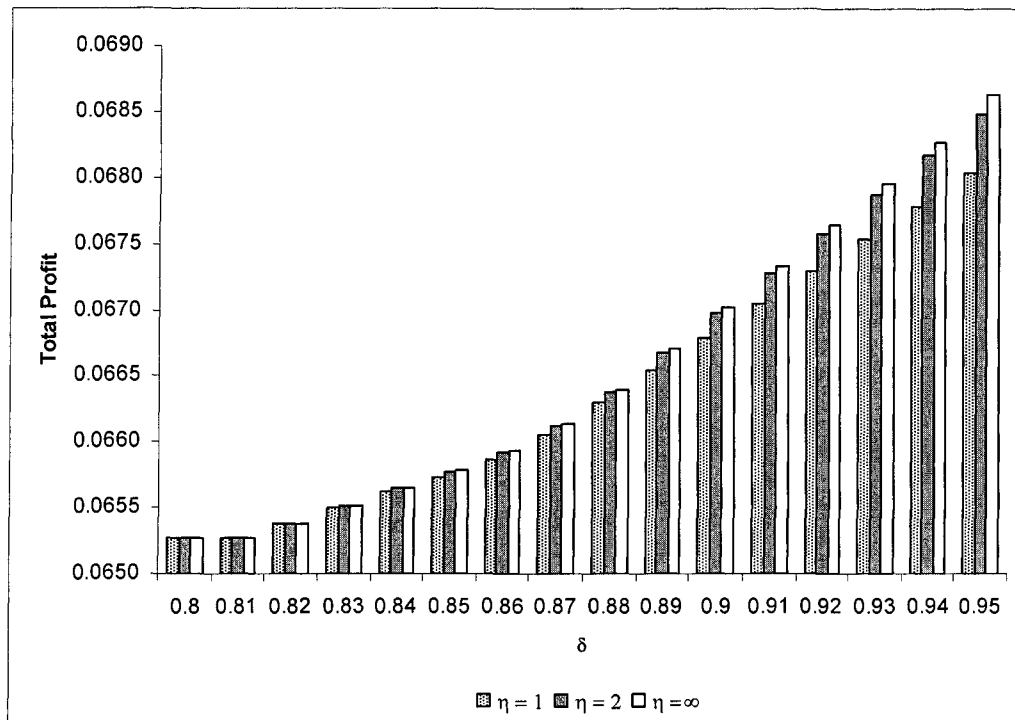
$$c_{34}^{(l,1)} = 0.04, c_{34}^{(l,2)} = 0.06, c_{34}^{(l,3)} = 0.10,$$

$$E[S_4^{(l,1)}] = 2.5, E[S_4^{(l,2)}] = 3.5, E[S_4^{(l,3)}] = 4.67.$$

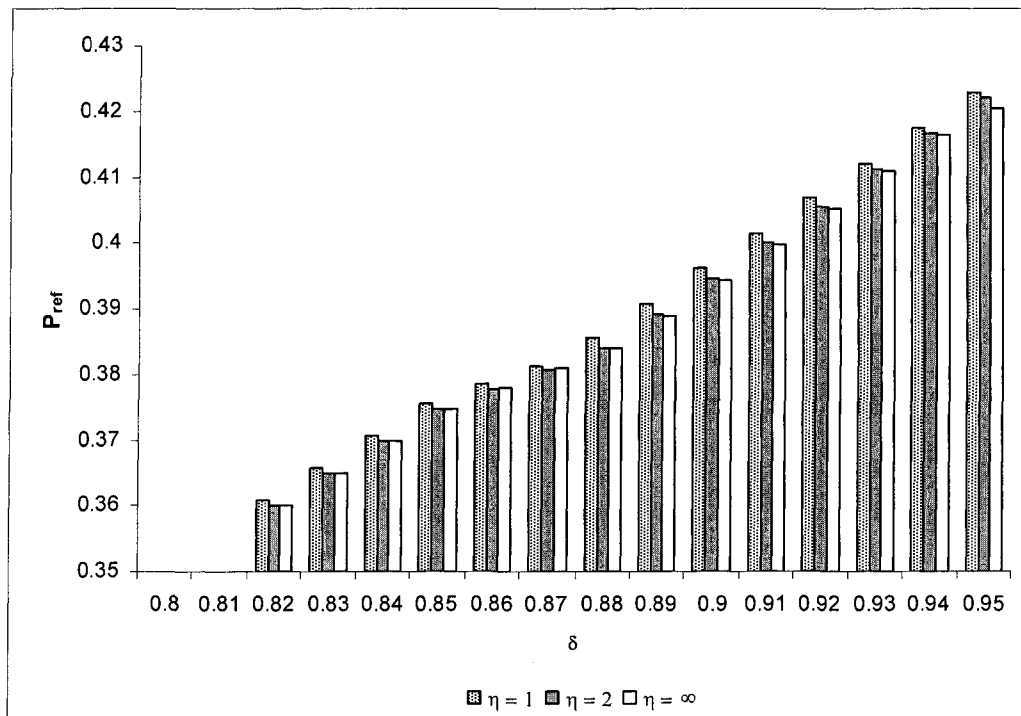
With these parameters, the results show no difference between grade and grade-type priority when the same quantity of products with the same grades are refurbished. For example, when the arrival rate to the refurbish station,  $\lambda_4 = 0.54355$ , the proportion of returned products to be refurbished in grade priority model,  $p_{mr}^{(\bullet 1)} = 0.1$ , yields  $P_{ref}^* = 0.404355$  and total profit = 0.065214. The grade-type priority model, with the same value of  $\lambda_4$ , yields the same  $P_{ref}^*$  and total profit at point  $p_{mr}^{(1,1)} = 0.101$ . Therefore, to compare the effect of the number of times products can be refurbished, we use the grade-priority model as a baseline.

Figure 20a shows that  $\eta$  has a positive effect on total profit when the optimal quantity of refurbished products is greater than zero. This effect increases with the quality of refurbished products. As for the decision variables, there is no difference in optimal price and quantity of refurbished products from  $\delta = 0.8$  to 0.81 since none of returned products are sent to be refurbished. Therefore,  $P_{ref}^*$  equals its upper bound resulting in zero demand for refurbished products as shown in Figures 20b and 20c. Unlike the previous results where  $P_{ref}^*$  decreases when  $p_{34}^*$  increases to match demand and supply of refurbished products, at higher quality levels, the relationship of  $P_{ref}^*$  and  $p_{34}^*$  is not straightforward. For  $\delta = 0.86$  to 0.88,  $p_{34}^*$  decreases as the value of  $\eta$  increase but  $P_{ref}^*$  decreases then increases as  $\eta$  changes from 1 to 2 to  $\infty$ . This behavior is not surprising because the higher proportion of products to refurbish does not necessarily mean the higher quantity of products to refurbish. For  $\delta = 0.86$ , although the value of  $p_{34}^*$  is higher at  $\eta = 1$  than at  $\eta = 2$ , the quantity of refurbished products is in fact lower.

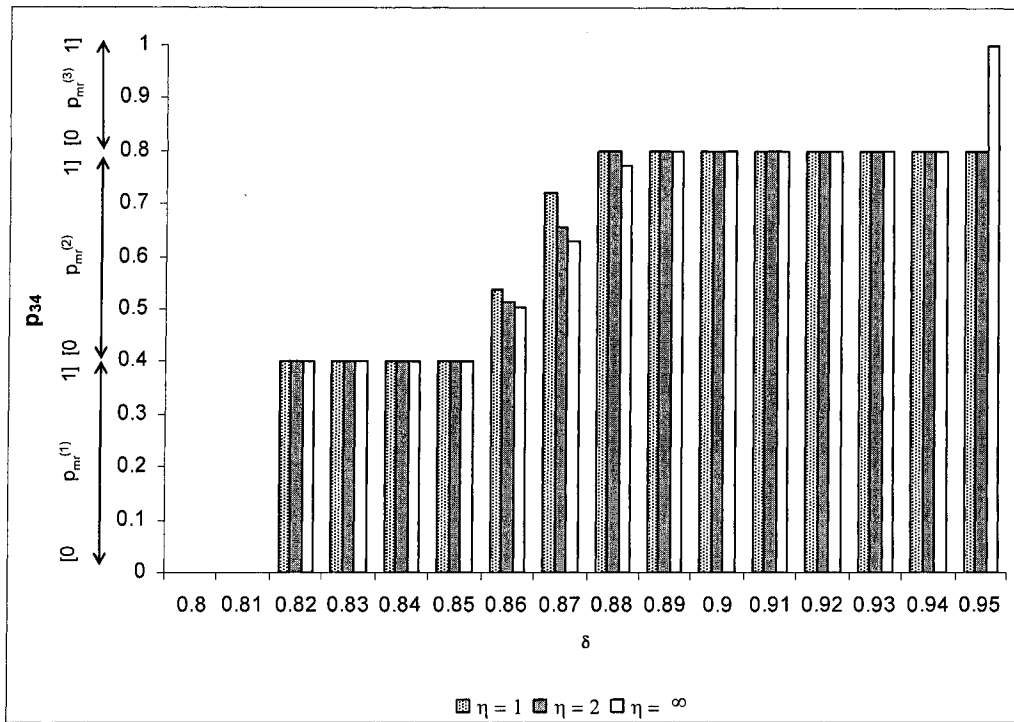
In summary, prioritizing returned products based on their return histories and their grade results in the same total profit as prioritizing only on their grades if the refurbishing cost, time and variability are not affected by the return history. Increasing the number of times products can be refurbished yields the higher total profit. However, the manufacturer should be aware of the additional cost of manufacturing and refurbishing products to be able to refurbish several times, and of the probability that products will be returned after purchase. The low value of  $p_{cr}$  will lower the amount of refurbishable products cycling in the network which results in a smaller improvement in total profit when  $\eta$  increases. The price of refurbished products is optimally placed to match the quantity of refurbished products ( $\lambda_4$ ) but not the proportion of returned products to be refurbished ( $p_{34}$ ), e.g.,  $P_{ref}$  increases when  $\lambda_4$  decreases and vice versa.



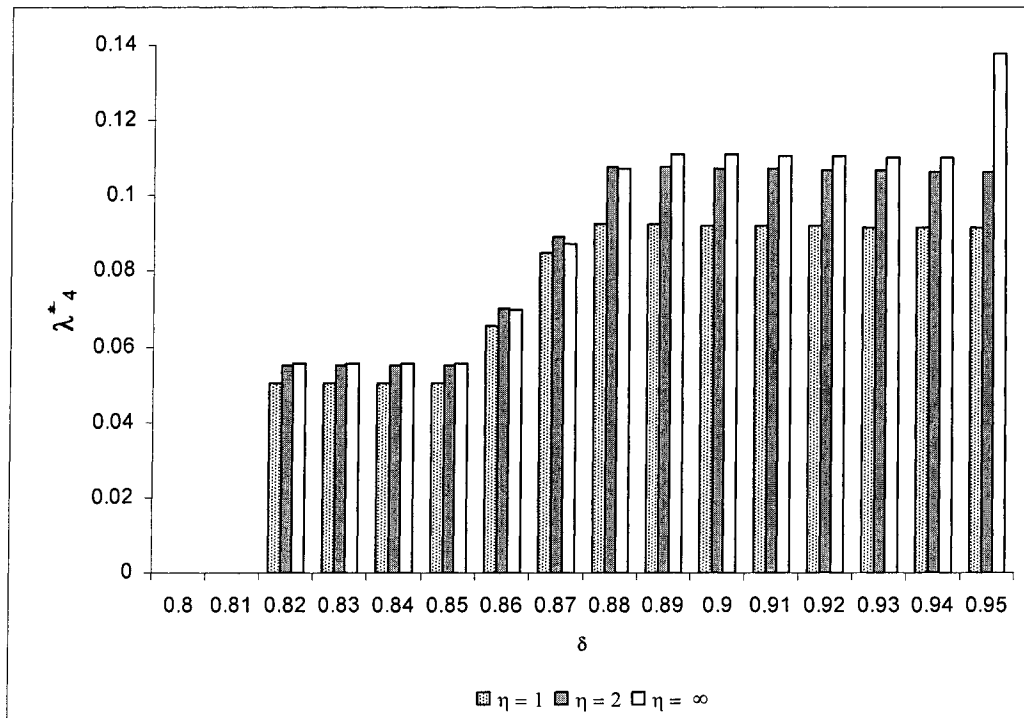
(a)



(b)



(c)



(d)

Figure 20. Total profit,  $P_{ref}^*$ ,  $p_{34}^*$  and  $\lambda_4^*$  from the general open queueing network with different numbers of times that products can be refurbished.

## CHAPTER 5. CONCLUSIONS AND FUTURE RESEARCH

This model confirms the results of other studies in suggesting that, for a manufacturer in a competitive market, introducing refurbished products to the market can be profitable even when they potentially reduce the demand for new products. The optimization model sets the appropriate price of refurbished products, relative to the new product price, and the proportion of product returns to be refurbished. It also shows how opening up a market for refurbished products may be necessary to relieve a capacity constraint for manufacturing new products. The numerical study reveals that significant proportions of returns should be refurbished when demand for new products and high backorder penalties combine with low refurbishment costs and high perceived quality of the refurbished products. The results show that the optimal price of refurbished products is placed corresponding to not only supply and demand of both new and refurbished products but also quality and several costs incurred from managing returned products.

The general queueing model allows us to model more realistic variability in the system. The variability in service times has an effect on inventory cost that causes changes in the optimal solution. When returned products come with distinct grades, evaluating and sorting products reduce the cost and time for refurbishing. This saving is significant when the manufacturer refurbishes only some of returns. Implementing the SEPT service protocol can also reduce inventory cost at the refurbishing stations as opposed to FCFS service protocol. At the point where refurbishing is profitable, increasing the number of times products can be refurbished improves the total profit.

We believe the application of our model is not limited to only the electronic industry, but can be applied in variety of products that are manufactured to order and refurbished to stock. As stated in Plambeck and Ward (2005), many companies such as General Electric, American Standard, BMW and Toyota are moving toward Make-to-Order approach. Moreover, automotive industries such as Honda, BMW and Toyota are already selling their certified pre-owned vehicles online. Although the model was motivated by electronics manufacturers who refurbish the products internally in their own facilities, the model could also be applied in the situation where product returns are refurbished by an external subcontractor as long as the objective remains to maximize the total profit from selling new, refurbished and dismantled products.

The observed discontinuous structure for the optimal policy suggests interesting avenues for further research. The optimality conditions and numerical results suggest that when manufacturing capacity is sufficient to meet demand, then it is optimal to either not refurbish any returns or to refurbish a significant proportion of them. Under certain conditions, very small changes in conditions will cause the decision to swing from one extreme to the other. Moreover, this characteristic is observed in the absence of either fixed costs for setting up the refurbishment operations or other economies of scale. The steady state assumption might be applied in only one interval of time when market size is rather constant. The model might not be suitable for the beginning or the end of product life cycle where demand and market size changes rapidly. It can be improved by assigning parameters such as market size, price and cost as functions of time. Further research is needed to understand the interactions among the different components of revenue and cost in this highly interdependent closed loop system.

Although the results show that higher numbers of times products can be refurbished yield higher profit, the manufacturer might need to invest additional cost in manufacturing and designing more durable products to be able to refurbish several times. In this research, we disregard the relationship of grades and number of returns. In fact, returned products from first time usage should have better grade distributions than those that have been refurbished several times. In the case where the proportions of grades depend on the return histories, what would be the optimal number times products can be refurbished?

Variabilities in the grades of product returns and consumer perceptions of the quality of refurbished products are also worth exploring further. Some returns might have as good as new quality and thus could be resold directly. On the other hand, some returns might be damaged so much that it is not worthwhile to refurbish them. With regard to quality, some consumers might be nearly indifferent between new and refurbished products while only new items would satisfy others. Appropriate modifications to the demand function could incorporate these differences.



## APPENDIX

### 1. The Feasible Region and its Convexity

Case 1. General open queueing network model with grades and number of returns priority

From section 3.2,

$$\begin{aligned}
 p_{34} &= \frac{\lambda_{34}}{\lambda_3} = \frac{\lambda_{new} \left( \sum_{l=1}^{\eta} \prod_{k=1}^l p_{cr} \sum_{r=1}^{\theta} (p_{mr}^{(k,r)} q^{(k,r)}) \right)}{\lambda_{new} p_{cr} \left( 1 + \sum_{l=1}^{\eta} \prod_{k=1}^l p_{cr} \sum_{r=1}^{\theta} (p_{mr}^{(k,r)} q^{(k,r)}) \right)} \\
 &= \frac{\sum_{l=1}^{\eta} \prod_{k=1}^l p_{cr} \sum_{r=1}^{\theta} (p_{mr}^{(k,r)} q^{(k,r)})}{p_{cr} \left( 1 + \sum_{l=1}^{\eta} \prod_{k=1}^l p_{cr} \sum_{r=1}^{\theta} (p_{mr}^{(k,r)} q^{(k,r)}) \right)}
 \end{aligned}$$

Let  $H$  and  $L$  be a set of products that have higher and lower priority than product  $(l,r)$  respectively from the priority scheme discussed in section 3.3. Then  $p_{mr}^{(k,s)} = 1 \forall k,s | (k,s) \in H$ ,  $p_{mr}^{(m,t)} = 0, \forall m,t | (m,t) \in L$ . Then  $p_{34}$  is a function of only  $p_{mr}^{(l,r)}$  and can be written as

$$p_{34} = \frac{R}{p_{cr}(1+R)} \text{ where}$$

$$R = \sum_{l=1}^{\eta} \left( \prod_{k=1}^l p_{cr} \sum_{g=1}^{\theta} q^{(k,g)} p_{mr}^{(k,g)} \right) \text{ such that } p_{mr}^{(k,s)} = 1 \forall k,s | (k,s) \in H, p_{mr}^{(m,t)} = 0,$$

$$\forall m,t | (m,t) \in L.$$

To illustrate  $R$ , we provide examples of  $R$  when  $\eta = 2, \theta = 3$  as follows:

Priority  $(1,1) \succ (2,1) \succ (1,2) \succ (2,2) \succ (1,3) \succ (2,3)$ :

$$\text{For } (l,r) = (1,1) : R = p_{cr} q^{(1,1)} p_{mr}^{(1,1)}.$$

For  $(l,r) = (2,1)$ :  $R = p_{cr}q^{(1,1)} + p_{cr}q^{(2,1)}p_{mr}^{(2,1)}p_{cr}q^{(1,1)}$ .

For  $(l,r) = (1,2)$ :  $R = p_{cr}(q^{(1,1)} + p_{mr}^{(1,2)}q^{(1,2)}) + p_{cr}(q^{(1,1)} + p_{mr}^{(1,2)}q^{(1,2)})p_{cr}q^{(2,1)}$ .

For  $(l,r) = (2,2)$ :  $R = p_{cr}(q^{(1,1)} + q^{(1,2)}) + p_{cr}(q^{(1,1)} + q^{(1,2)})p_{cr}(q^{(2,1)} + q^{(2,2)}p_{mr}^{(2,2)})$ .

For  $(l,r) = (1,3)$ :

$$R = p_{cr}(q^{(1,1)} + q^{(1,2)} + q^{(1,3)}p_{mr}^{(1,3)}) + p_{cr}(q^{(1,1)} + q^{(1,2)} + q^{(1,3)}p_{mr}^{(1,3)})p_{cr}(q^{(2,1)} + q^{(2,2)})$$

For  $(l,r) = (2,3)$ :

$$R = p_{cr}(q^{(1,1)} + q^{(1,2)} + q^{(1,3)}) + p_{cr}^2(q^{(1,1)} + q^{(1,2)} + q^{(1,3)})p_{cr}(q^{(2,1)} + q^{(2,2)} + q^{(2,3)}p_{mr}^{(2,3)})$$

We clearly see that  $p_{34}$  is a nonlinear function of  $p_{mr}^{(l,r)}$ . For each product class  $(l,r)$ , constraints (1) to (9) can be expressed in the form of  $g_k(P_{ref}, p_{34}(p_{mr}^{(l,r)})) - b_k \leq 0$ ,  $k = 1, \dots, 9$

as follows

$$-P_{ref} + P_{new} - (1 - \delta) \leq 0 \quad (1a)$$

$$P_{ref} - \delta P_{new} \leq 0 \quad (2a)$$

$$P_{ref} - (P_{new} - (1 - \delta)(1 - (1 - \varepsilon_1)\mu_1)) \leq 0 \quad (3a)$$

$$P_{ref} - \frac{(1 - \delta)(1 - \varepsilon_3)\mu_3 + p_{cr}\{P_{new} - (1 - \delta)(1 + (1 - \varepsilon_3)\mu_3 p_{34}(p_{mr}^{(l,r)}))\}}{p_{cr}} \leq 0 \quad (4a)$$

$$P_{ref} - \frac{(1 - \delta)(1 - \varepsilon_4)\mu_4 + p_{cr}\{P_{new} - (1 - \delta)(1 + (1 - \varepsilon_4)\mu_4)\}p_{34}(p_{mr}^{(l,r)})}{p_{cr}p_{34}(p_{mr}^{(l,r)})} \leq 0 \quad (5a)$$

$$P_{ref} - \frac{\delta(1 - \varepsilon_5)P_{new} - p_{cr}\delta(1 - \delta - \varepsilon_5 P_{new})p_{34}(p_{mr}^{(l,r)})}{1 - \varepsilon_5(1 - p_{cr}p_{34}(p_{mr}^{(l,r)})) - p_{cr}(1 - \delta)p_{34}(p_{mr}^{(l,r)})} \leq 0 \quad (6a)$$

$$\frac{\delta(\gamma P_{new} - p_{cr}(1 - P_{new}(1 - \gamma) - \delta))p_{34}(p_{mr}^{(l,r)})}{\gamma(1 - p_{cr}p_{34}(p_{mr}^{(l,r)})) + \delta p_{cr}p_{34}(p_{mr}^{(l,r)})} - P_{ref} \leq 0 \quad (7a)$$

$$p_{mr}^{(l,r)} - 1 \leq 0 \quad (8a)$$

$$-p_{mr}^{(l,r)} \leq 0 \quad (9a)$$

where  $\varepsilon_k$ ,  $k = 1, 3, 4, 5$  are some small positive constants.

Referring to section 3.3, feasible solutions are not limited by (4a) and (5a). Therefore, only (3a), (6a), (7a), (8a) and (9a) can be boundaries of the feasible region. From Rardin (1998), the feasible set defined by constraints  $g_k(P_{ref}, p_{mr}^{(l,r)}) - b_k \leq 0$ ,  $k = 1, \dots, 9$  is convex if each  $g_k$  is convex function. Since  $g_k(P_{ref}, p_{mr}^{(l,r)})$ 's are separable, we can consider the second derivative of each variable individually. The functions  $g_k$  for  $k = 3, 8$  and  $9$  are linear and therefore convex. Constraints (6a) and (7a) are convex if  $\frac{\partial^2 g_k}{\partial p_{mr}^{(l,r)^2}} \geq 0$ .

The second derivative for each  $(l, r)$  of  $g_k(P_{ref}, p_{mr}^{(l,r)})$ ,  $k = 6, 7$  can be obtained as follows;

$$\begin{aligned} \text{Let } R' &= \frac{\partial}{\partial p_{mr}^{(l,r)}} \left( \sum_{\forall s|(l,s) \in H} q^{(l,s)} + q^{(l,r)} p_{mr}^{(l,r)} \right) \sum_{\forall y|y \neq l} \prod_{k=1}^y p_{cr} \sum_{\forall s|(k,s) \in H} q^{(k,s)} \\ &= q^{(l,r)} \sum_{\forall y|y \neq l} \prod_{k=1}^y p_{cr} \sum_{\forall s|(k,s) \in H} q^{(k,s)} \geq 0, \end{aligned}$$

$$R'' = \frac{\partial^2}{\partial p_{mr}^{(l,r)^2}} \left( \sum_{\forall s|(l,s) \in H} q^{(l,s)} + q^{(l,r)} p_{mr}^{(l,r)} \right) \sum_{\forall y|y \neq l} \prod_{k=1}^y p_{cr} \sum_{\forall s|(k,s) \in H} q^{(k,s)} = 0. \text{ Then from the chain rule}$$

$$\frac{\partial p_{34}}{\partial p_{mr}^{(l,r)}} = \frac{\partial p_{34}}{\partial R} R' = \frac{R'}{p_{cr}(1+R)^2} \geq 0.$$

$$\frac{\partial^2 p_{34}}{\partial p_{mr}^{(l,r)^2}} = \frac{\partial^2 p_{34}}{\partial R^2} (R')^2 + \frac{\partial p_{34}}{\partial R} R'' = \frac{\partial^2 p_{34}}{\partial R^2} (R')^2 = -\frac{2(R')^2}{p_{cr}(1+R)^3} \leq 0$$

$$\frac{\partial g_6}{\partial p_{mr}^{(l,r)}} = \frac{\partial g_6}{\partial p_{34}} \frac{\partial p_{34}}{\partial p_{mr}^{(l,r)}} = \frac{p_{cr}(1-P_{new})(1-\delta)\delta(1-\varepsilon_5)}{(1-\varepsilon_5(1-p_{cr}p_{34})-p_{cr}p_{34}(1-\delta))^2} \frac{R'}{p_{cr}(1+R)^2} \geq 0$$

$$\begin{aligned} \frac{\partial^2 g_6}{\partial p_{mr}^{(l,r)^2}} &= \frac{\partial^2 g_6}{\partial p_{34}^2} \left( \frac{\partial p_{34}}{\partial p_{mr}^{(l,r)}} \right)^2 + \frac{\partial g_6}{\partial p_{34}} \frac{\partial^2 p_{34}}{\partial p_{mr}^{(l,r)^2}} \\ &= \frac{2p_{cr}^2(1-P_{new})(1-\delta)\delta(1-\varepsilon_5)(1-\delta-\varepsilon_5)}{(1-\varepsilon_5(1-p_{cr}p_{34})-p_{cr}p_{34}(1-\delta))^3} \left( \frac{R'}{p_{cr}(1+R)^2} \right)^2 + \\ &\quad \frac{p_{cr}(1-P_{new})(1-\delta)\delta(1-\varepsilon_5)}{(1-\varepsilon_5(1-p_{cr}p_{34})-p_{cr}p_{34}(1-\delta))^2} \left( -\frac{2R'^2}{p_{cr}(1+R)^3} \right) \\ &= R'^2 \frac{2p_{cr}^2(1-P_{new})(1-\delta)\delta(1-\varepsilon_5)((1-\delta-\varepsilon_5)-(1+R)(1-\varepsilon_5(1-p_{cr}p_{34})-p_{cr}p_{34}(1-\delta)))}{p_{cr}^2(1+R)^4(1-\varepsilon_5(1-p_{cr}p_{34})-p_{cr}p_{34}(1-\delta))^3} \end{aligned}$$

Since  $p_{cr}p_{34} = \frac{R}{1+R}$ ,  $(1+R)(1-\varepsilon_5(1-p_{cr}p_{34})-p_{cr}p_{34}(1-\delta)) = (1+R\delta-\varepsilon_5)$ .

Therefore

$$\begin{aligned} \frac{\partial^2 g_6}{\partial p_{mr}^{(l,r)^2}} &= R'^2 \frac{2p_{cr}^2(1-P_{new})(1-\delta)\delta(1-\varepsilon_5)((1-\delta-\varepsilon_5)-(1+R\delta-\varepsilon_5))}{p_{cr}^2(1+R)(1+R\delta-\varepsilon_5)^3} \\ &= -\frac{2R'^2(1-P_{new})(1-\delta)\delta^2(1-\varepsilon_5)}{(1+R\delta-\varepsilon_5)^3} \leq 0 \end{aligned}$$

$$\begin{aligned} \frac{\partial^2 g_7}{\partial p_{mr}^{(l,r)^2}} &= \frac{\partial^2 g_7}{\partial p_{34}^2} \left( \frac{\partial p_{34}}{\partial p_{mr}^{(l,r)}} \right)^2 + \frac{\partial g_7}{\partial p_{34}} \frac{\partial^2 p_{34}}{\partial p_{mr}^{(l,r)^2}} \\ &= \frac{2p_{cr}^2(1-P_{new})(\delta-\gamma)\gamma(1-\delta)}{(1-p_{cr}p_{34}(\delta-\gamma))^3} \left( \frac{R'}{p_{cr}(1+R)^2} \right)^2 \\ &\quad - \frac{p_{cr}\gamma\delta(1-P_{new})(1-\delta)}{((1-p_{cr}p_{34})\gamma+p_{cr}p_{34}\delta)^2} \left( -\frac{2R'^2}{p_{cr}(1+R)^3} \right) \end{aligned}$$

$$= \frac{2R'^2(1-P_{new})(1-\delta)\delta^2\gamma}{(\gamma+R\delta)^3} \geq 0$$

Because  $g_6$  is concave rather than convex, the feasible region in the general open queueing network model with grades and number of returns priority is not convex.

*Case 2. General open queueing network model with grade priority with  $\eta \rightarrow \infty$*

Let  $H$  and  $L$  be the sets of products that have higher and lower priority than product ( $r$ ). Then from the priority discussed in section 3.3,  $p_{mr}^{(s)} = 1 \quad \forall s | s \in H$ ,  $p_{mr}^{(t)} = 0$ ,  $\forall t | t \in L$ . Then  $p_{34}$  is the function of only  $p_{mr}^{(r)}$  and can be written by  $p_{34} = q^{(r)}p_{mr}^{(r)} + \sum_{s \in H} q^{(s)}$ , where  $\sum_{s \in H} q^{(s)}$  is the summation of proportions of returned products that have higher grade priority than  $r$ . For each product grade  $r$ , constraints (1) to (9) can be expressed as functions of  $g_k(P_{ref}, p_{34}(p_{mr}^{(r)})) - b_k \leq 0$ ,  $k = 1, \dots, 9$  as follows:

$$-P_{ref} + P_{new} - (1-\delta) \leq 0 \quad (1b)$$

$$P_{ref} - \delta P_{new} \leq 0 \quad (2b)$$

$$P_{ref} - (P_{new} - (1-\delta)(1 - (1-\varepsilon_1)\mu_1)) \leq 0 \quad (3b)$$

$$P_{ref} - \frac{(1-\delta)(1-\varepsilon_3)\mu_3 + p_{cr} \{P_{new} - (1-\delta)(1 + (1-\varepsilon_3)\mu_3 p_{34}(p_{mr}^{(r)})\}}}{p_{cr}} \leq 0 \quad (4b)$$

$$P_{ref} - \frac{(1-\delta)(1-\varepsilon_4)\mu_4 + p_{cr} \{P_{new} - (1-\delta)(1 + (1-\varepsilon_4)\mu_4)\} p_{34}(p_{mr}^{(r)})}{p_{cr} p_{34}(p_{mr}^{(r)})} \leq 0 \quad (5b)$$

$$P_{ref} - \frac{\delta(1-\varepsilon_5)P_{new} - p_{cr} \delta(1-\delta - \varepsilon_5 P_{new}) p_{34}(p_{mr}^{(r)})}{1-\varepsilon_5(1-p_{cr} p_{34}(p_{mr}^{(r)})) - p_{cr}(1-\delta) p_{34}(p_{mr}^{(r)})} \leq 0 \quad (6b)$$

$$\frac{\delta(\gamma P_{new} - p_{cr}(1 - P_{new}(1 - \gamma) - \delta)p_{34}(p_{mr}^{(\bullet r)}))}{\gamma(1 - p_{cr}p_{34}(p_{mr}^{(\bullet r)})) + \delta p_{cr}p_{34}(p_{mr}^{(\bullet r)})} - P_{ref} \leq 0 \quad (7b)$$

$$p_{mr}^{(\bullet r)} - 1 \leq 0 \quad (8b)$$

$$-p_{mr}^{(\bullet r)} \leq 0 \quad (9b)$$

where  $\varepsilon_k, k = 1, 3, 4, 5$  are some small positive constants.

From case 1, (6a) and (7a) are convex if  $\frac{\partial^2 g_6}{\partial p_{mr}^{(\bullet r)2}}$  and  $\frac{\partial^2 g_7}{\partial p_{mr}^{(\bullet r)2}}$  are nonnegative. We know that

$\frac{\partial^2 g_6}{\partial p_{34}^2}$  is positive and  $\frac{\partial^2 g_7}{\partial p_{34}^2}$  is also positive when  $\delta > \gamma$ . Therefore the sign of the second

derivative of  $g_6$  and  $g_7$  depends only on  $\frac{\partial g_6}{\partial p_{34}} \frac{\partial^2 p_{34}}{\partial p_{mr}^{(\bullet r)2}}$  and  $\frac{\partial g_7}{\partial p_{34}} \frac{\partial^2 p_{34}}{\partial p_{mr}^{(\bullet r)2}}$ . However  $\frac{\partial^2 p_{34}}{\partial p_{mr}^{(\bullet r)2}} =$

0 because  $p_{34}$  is a linear function of  $p_{mr}^{(\bullet r)}$ . By the assumption that  $\gamma$  is small, the feasible region of general open queueing network model with grade priority is convex.

*Case 3. General open queueing network with no priority and Jackson open queueing network model*

These two models have only one class of product,  $p_{34} = p_{mr}$  so that  $\frac{\partial^2 p_{34}}{\partial p_{mr}^2} = 0$ .

Constraints (1) to (9) can be expressed as function of  $g_k(p_{ref}, p_{mr}) - b_k \leq 0, k = 1, \dots, 9$  as

follows

$$-P_{ref} + P_{new} - (1 - \delta) \leq 0 \quad (1c)$$

$$P_{ref} - \delta P_{new} \leq 0 \quad (2c)$$

$$P_{ref} - (P_{new} - (1 - \delta)(1 - (1 - \varepsilon_1)\mu_1)) \leq 0 \quad (3c)$$

$$P_{ref} - \frac{(1 - \delta)(1 - \varepsilon_3)\mu_3 + p_{cr}(P_{new} - (1 - \delta)(1 + (1 - \varepsilon_3)\mu_3 p_{mr}))}{p_{cr}} \leq 0 \quad (4c)$$

$$P_{ref} - \frac{(1 - \delta)(1 - \varepsilon_4)\mu_4 + p_{cr}(P_{new} - (1 - \delta)(1 + (1 - \varepsilon_4)\mu_4))p_{mr}}{p_{cr}p_{mr}} \leq 0 \quad (5c)$$

$$P_{ref} - \frac{\delta(1 - \varepsilon_5)P_{new} - p_{cr}\delta(1 - \delta - \varepsilon_5 P_{new})p_{mr}}{1 - \varepsilon_5(1 - p_{cr}p_{mr}) - p_{cr}(1 - \delta)p_{mr}} \leq 0 \quad (6c)$$

$$\frac{\delta(\gamma P_{new} - p_{cr}(1 - P_{new}(1 - \gamma) - \delta)p_{mr})}{\gamma(1 - p_{cr}p_{mr}) + p_{cr}p_{mr}\delta} - P_{ref} \leq 0 \quad (7c)$$

$$p_{mr} - 1 \leq 0 \quad (8c)$$

$$-p_{mr} \leq 0 \quad (9c)$$

where  $\varepsilon_k, k = 1, 3, 4, 5$  are some small positive constants.

Similar to case 2,  $\frac{\partial^2 P_{34}}{\partial p_{mr}^2} = 0$ , therefore the feasible region is convex when  $\delta > \gamma$ . By the

assumption that  $\gamma$  is small, the feasible regions in the general open queueing network with no priority and the Jackson open queueing network model are convex.

## 2. Derivation of KKT Conditions

Since constraints (6) and (7) are nonlinear, the problem has a nonlinear objective function and nonlinear inequality constraints. To help find its solution, we apply the Karush-Kuhn-Tucker (KKT) optimality conditions (Bazaraa et al., 1993). Introducing the Lagrange multipliers  $u_k, k = 3, 6, 7, 8, 9$ , if  $(P_{ref}, p_{mr}^{(l,r)})$  is the feasible solution, the conditions for a maximization problem are:

1. The gradient of the Lagrangian function equals zero.

$$-\frac{\partial f(P_{ref}^*, P_{mr}^{(l,r)*})}{\partial P_{ref}} + u_3 + u_6 - u_7 = 0$$

$$-\frac{\partial f(P_{ref}^*, P_{mr}^{(l,r)*})}{\partial P_{mr}^{(l,r)}} + u_6 \frac{\partial}{\partial P_{mr}^{(l,r)}} g_6(P_{ref}^*, P_{mr}^{(l,r)*}) - u_7 \frac{\partial}{\partial P_{mr}^{(l,r)}} g_7(P_{ref}^*, P_{mr}^{(l,r)*}) + u_8 - u_9 = 0$$

2. The constraints and multipliers satisfy complementary slackness conditions.

$$u_3 (P_{ref}^* - P_{new} + (1 - \delta)(1 - (1 - \varepsilon_1)\mu_1)) = 0$$

$$u_6 g_6(P_{ref}^*, P_{mr}^{(l,r)*}) = 0$$

$$u_7 g_7(P_{ref}^*, P_{mr}^{(l,r)*}) = 0$$

$$u_8 (P_{mr}^{(l,r)*} - 1) = 0$$

$$u_9 (-P_{mr}^{(l,r)*}) = 0$$

3. The Lagrange multipliers are nonnegative.

$$u_k \geq 0, k = 3, 6, 7, 8, 9$$

These conditions are necessary for optimality if a set of constraint qualifications is satisfied.

Winston (2004) provides a simple set of constraint qualifications: Let  $(P_{ref}^*, P_{mr}^{(l,r)*})$  be an optimal solution. If all the constraints are continuous, and the gradients of all binding constraints at  $(P_{ref}^*, P_{mr}^{(l,r)*})$  form a set of linearly independent vectors, then the KKT conditions must hold at  $(P_{ref}^*, P_{mr}^{(l,r)*})$ . These qualifications are clearly satisfied by our constraints because the only pair of gradients that are not linearly independent are those for (8) and (9); however, these two constraints cannot be binding simultaneously.



If  $P_{new} \leq 1 - \mu_1$ , then constraint (3a) places a more restrictive upper bound on  $P_{ref}$  than does (6a). In this case, for the feasible region to be nonempty, the curves corresponding to (3a) and (7a) must cross within the bounds of the other constraints. This occurs when  $\gamma < \frac{p_{cr}\delta\mu_1}{(1-p_{cr})(1-P_{new}-\mu_1)}$ . We assume that  $\gamma$  is quite small, in which case the boundary from constraint (7a) is only slightly more restrictive than the other lower bounds on the decision variables provided by (1a) and (9a), respectively. Then constraint (7a) will be active only where its curve crosses the vertical axis or (3a), as described in case 1 below. Constraint (3a) and (6a) are equivalent to  $\rho_i \leq 1 - \varepsilon_i$ ,  $i = 1$  and  $5$ . When constraints (3a) and (6a) are active,  $\rho_i = 1 - \varepsilon_i$ ,  $i = 1$  and  $5$ , and the backorder cost at station 1 and holding cost at station 5 are  $h_i E[N_i] = h_i \left( \frac{\rho_i}{1-\rho_i} \right) = h_i \left( \frac{1-\varepsilon_i}{\varepsilon_i} \right)$ ,  $i = 1$  and  $5$ , respectively. Since  $\varepsilon_i$  is arbitrarily small, the cost at station 1 and 5 will be too high for the maximum profit to be achieved. Therefore (3a) and (6a) will not be active at optimal solution, i.e.,  $u_3 = u_6 = 0$ .

The KKT conditions are reduced to the following set:

$$-\frac{\partial f(P_{ref}^*, p_{mr}^{(l,r)*})}{\partial P_{ref}} - u_7 = 0 \quad (10a)$$

$$-\frac{\partial f(P_{ref}^*, p_{mr}^{(l,r)*})}{\partial p_{mr}^{(l,r)}} - u_7 \frac{\partial}{\partial p_{34}} g_7(P_{ref}^*, p_{34}^*) \frac{\partial p_{34}}{\partial p_{mr}^{(l,r)}} + u_8 - u_9 = 0 \quad (11a)$$

$$u_7 g_7(P_{ref}^*, p_{mr}^{(l,r)*}) = 0 \quad (12a)$$

$$u_8 (p_{mr}^{(l,r)*} - 1) = 0 \quad (13a)$$

$$u_9 (-p_{mr}^{(l,r)*}) = 0 \quad (14a)$$

$$u_k \geq 0, k = 7, 8, 9$$

In terms of values of  $p_{mr}^{(l,r)}$ , three possible cases of local optima are:

1) The solution lies on the minimum  $p_{mr}^{(l,r)}$ : When  $P_{new} > 1 - \mu_1$ , the minimum is  $p_{34} = 0$

or where (7a) crosses the vertical axis (Figure 5b). At this point,  $u_8 = 0$  and

$u_7, u_9 \geq 0$ . If  $P_{new} \leq 1 - \mu_1$ , the minimum  $p_{34}(p_{mr}^{(l,r)}) =$

$$\frac{\gamma(1 - P_{new} - \mu_1(1 - \varepsilon_1))}{p_{cr}(\delta\mu_1(1 - \varepsilon_1) + \gamma(1 - P_{new} - \mu_1(1 - \varepsilon_1)))} > 0, \text{ where (3a) and (7a) cross (Figure 5a). At}$$

this point,  $u_8 = u_9 = 0$ , and  $u_7 \geq 0$  along with conditions (10a) and (11a) imply that

$$\nabla f(p_{ref}, p_{mr}) \leq 0.$$

2) The solution lies between the  $p_{mr}$  from case 1 and  $p_{mr} = 1$ . Assuming a minimal

value of  $\gamma$ , no constraints are active, and the KKT conditions imply that

$$\nabla f(p_{ref}, p_{34}(p_{mr}^{(l,r)})) = 0, \text{ i.e., a stationary point.}$$

3) The solution lies on  $p_{34} = 1$ . At this point,  $\frac{\partial f(p_{ref}^*, p_{mr}^{(l,r)*})}{\partial P_{ref}} \leq 0$  while

$$\frac{\partial f(p_{ref}^*, p_{mr}^{(l,r)*})}{\partial p_{mr}^{(l,r)}} \geq 0 \text{ for small } \gamma.$$

This KKT conditions for general open queueing network with grade and number of returns priority can be applied to other open queueing network models in this paper by replacing the

$$p_{mr}^{(l,r)} \text{ and } \frac{\partial}{\partial p_{34}} g_7(p_{ref}^*, p_{34}^*) \frac{\partial p_{34}}{\partial p_{mr}^{(l,r)}} \text{ by } p_{mr}^{(*r)} \text{ and } \frac{\partial}{\partial p_{34}} g_7(p_{ref}^*, p_{34}^*) \frac{\partial p_{34}}{\partial p_{mr}^{(*r)}}$$

queueing network model with grade priority and by  $p_{mr}$  and  $\frac{\partial}{\partial p_{mr}} g_7(p_{ref}^*, p_{mr}^*)$  for the general open queueing network with no priority and Jackson open queueing network model.

### 3. The concavity of total profit with respect to $P_{new}$

$$\begin{aligned} \frac{\partial^2}{\partial P_{new}^2} (\text{Total Profit}) &= \frac{\partial^2}{\partial P_{new}^2} (\text{Revenue} - \text{Total Cost}) \\ &= -\frac{2(1-p_{cr})}{1-\delta} - \frac{h_5 \partial^2 E[N_5]}{\partial P_{new}^2} \end{aligned}$$

$$\frac{\partial^2 E[N_5]}{\partial \rho_5^2} = \frac{2}{(1-\rho_5)^2} + \frac{2\rho_5}{(1-\rho_5)^3} \geq 0$$

$$\frac{\partial^2 \rho_5}{\partial P_{new}^2} = \frac{2p_{cr}p_{34} \left(1 - \frac{P_{new}-P_{ref}}{1-\delta}\right)}{(1-p_{cr}p_{34}) \left(\frac{P_{new}-P_{ref}}{1-\delta} - \frac{P_{ref}}{\delta}\right)^3 (1-\delta)^2} + \frac{2p_{cr}p_{34}}{(1-p_{cr}p_{34}) \left(\frac{P_{new}-P_{ref}}{1-\delta} - \frac{P_{ref}}{\delta}\right)^2 (1-\delta)^2} \geq 0$$

From the following properties of convex function Floudas (2000);

i) if  $f(x)$  is convex, then  $-f(x)$  is concave,

ii) if  $f_1(x), \dots, f_n(x)$  are concave functions on a convex subset  $S$  of  $R^n$ , then

$\sum_{i=1}^n f_i(x)$  is concave, and

iii) if  $f(x)$  is convex on a convex subset  $S$  of  $R^n$ , and  $g(x)$  is an increasing convex function defined on the range of  $f(x)$  in  $R$ , then the composite function  $g(f(x))$  is convex on  $S$ .

Since  $E[N_5]$  is an increasing convex function of  $\rho_5$  and  $\rho_5$  is a convex function of  $P_{new}$ ,  $E[N_5]$  is a convex function of  $P_{new}$ . We can conclude that total profit is a concave function of  $P_{new}$ .

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